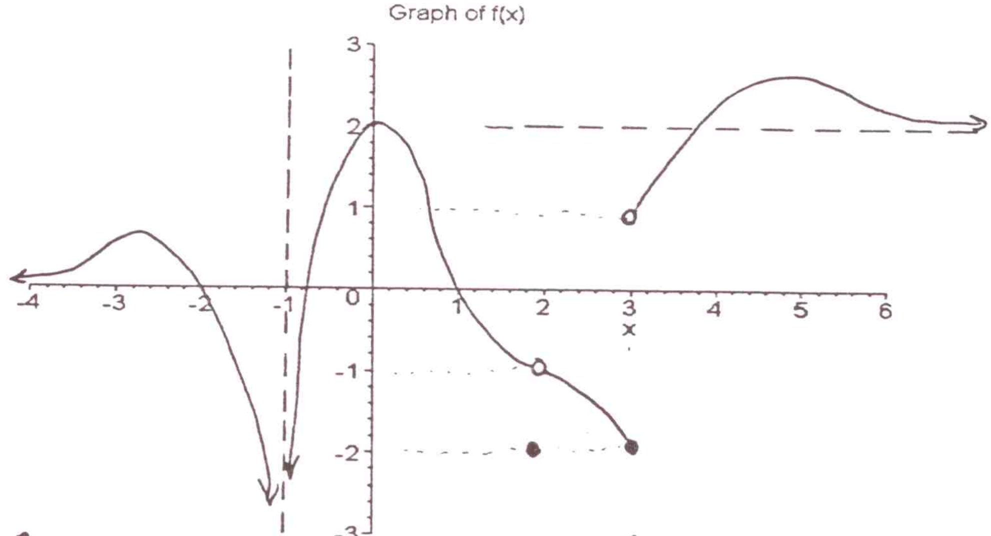
9/20/2000
Dr. Lunsford

MA303 Calculus I
Test 1

Name: Solutions
(100 Points Total)

I. Use the graph of the function f below to answer the following questions. (2 points each – 30 total)

Graph of f(x)



$f(1) = \underline{0}$ $\quad f(2) = \underline{-2}$ $\quad f(3) = \underline{-2}$

$\lim_{x\to\infty} f(x) = \underline{2}$ $\quad \lim_{x\to-\infty} f(x) = \underline{0}$ $\quad \lim_{x\to-1} f(x) = \underline{-\infty}$ $\quad \lim_{x\to 0} f(x) = \underline{2}$

$\lim_{x\to 3^+} f(x) = \underline{1}$ $\quad \lim_{x\to 2^-} f(x) = \underline{-1}$ $\quad \lim_{x\to 3} f(x) = \underline{DNE}$ $\quad \lim_{x\to -2} f(x) = \underline{0}$

For the remaining questions, please write "true" or "false", according to which is correct about the statement, in the space provided next to each statement.

False f is continuous at $x = 2$.
True f is continuous on the interval $(2,3]$.
True f is continuous from the left at $x = 3$.
False f is continuous on the interval $[3,\infty)$.

II. Complete the table below to find the value of the given limit. Show all decimal places on your calculator. (5 points)

$$\lim_{x\to 0}\frac{\sin x}{3x} = \frac{1}{3} = .\overline{3}$$

x	-0.1	-0.01	-0.001	.001	.01	.1
$f(x)$	.33277806	.33332778	.33333328	.33333328	.33332778	.33277806

III. Below you are given the graphs of $x^2 - y = 3$ and $x - y = 1$. (10 points total)

(a) Clearly indicate on the graph below which graph represents which function. (1 point)

(b) Find all x and y intercepts of the two graphs and label these points on the graphs below. You must show your work that justifies your answers. Clearly indicate your answers. (5 points)

(c) Find all intersection points of the two graphs and label these points on the graphs below. You must show your work that justifies your answers. Clearly indicate your answers. (4 points)

(b) $x^2 - y = 3$:

x-inter: $x^2 - 3 = 0$

$x = \pm\sqrt{3}$

y-inter: $-y = 3$

$y = -3$

$x - y = 1$

x inter: $x - 0 = 1$

$x = 1$

y inter: $-y = 1$

$y = -1$

(c) $x^2 - y = 3$

$x - y = 1 \Rightarrow y = x - 1$

$x^2 - (x-1) = 3$

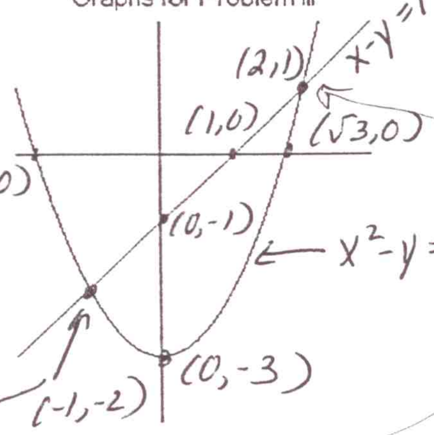
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\Rightarrow y = 1$ $\quad y = -2$

Graphs for Problem III

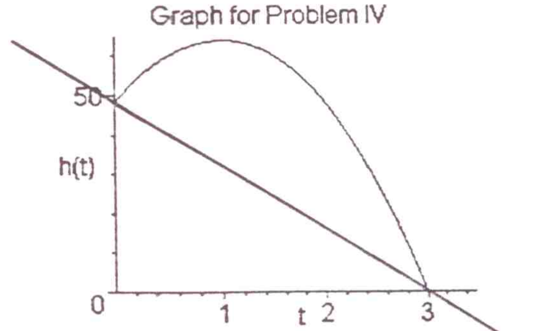


IV. At time $t = 0$ seconds a diver jumps from a 48 foot high diving board. The height of the diver (in feet) at time t seconds is given by $h(t) = -16t^2 + 32t + 48$. Find the average rate of change of the diver's height from $t = 0$ to $t = 3$ seconds. Draw the line on the graph below whose slope represents this average rate of change. (5 points)

$$\frac{\Delta h}{\Delta t} = \frac{h(3) - h(0)}{3 - 0}$$

$$= \frac{0 - 48}{3 - 0} = \boxed{-16 \text{ ft/s}}$$

Graph for Problem IV



V. Let $f(x) = \begin{cases} x^2 - 3x - 1, & x \geq 1 \\ \dfrac{x-7}{2x}, & x < 1 \end{cases}$. Determine if f is continuous at $x = 1$. You must show all work to justify your answer! Clearly indicate your answer. (7 points)

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 - 3x - 1 = -3$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x-7}{2x} = -3$

$\therefore \lim_{x \to 1} f(x) = 3$

$f(1) = (1)^2 - 3(1) - 1 = -3$

Yes, f is continuous at $x = 1$ since $\lim_{x \to 1} f(x) = -3 = f(1)$.

VI. Let $f(x) = x^2 - 3x + 1$. Find $\lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x}$. (8 points)

$$\lim_{\Delta x \to 0} \frac{(2+\Delta x)^2 - 3(2+\Delta x) + 1 - (-1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4 + 4\Delta x + (\Delta x)^2 - 6 - 3\Delta x + 2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 1 + \Delta x = \boxed{1}$$

VII. Find the indicated limits. Please show all work to justify your answers. (5 points each, 35 total)

1. $\lim_{x \to 3^-} \frac{3-x}{x^2 - 6x + 9} = \lim_{x \to 3^-} \frac{3-x}{(x-3)^2} = \lim_{x \to 3^-} \frac{-1}{x-3} = \boxed{+\infty}$

0, 0, 0^-

2. $\lim_{x \to \infty} \frac{(1 - 2x - 3x^2)\left(\frac{1}{x^2}\right)}{(2x^2 - 7x + 9)\left(\frac{1}{x^2}\right)} = \lim_{x \to \infty} \frac{\frac{1}{x^2} - \frac{2}{x} - 3}{2 - \frac{7}{x} + \frac{9}{x^2}} = \boxed{-\frac{3}{2}}$

∞, ∞

3. $\lim_{x \to 0} \frac{x^2}{\sin x} = \lim_{x \to 0} x \cdot \frac{x}{\sin x} = \lim_{x \to 0} x \cdot \frac{1}{\frac{\sin x}{x}} = \boxed{0}$

0, 0, 1

Or $\lim_{x \to 0} \frac{x^2}{\sin x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to 0} \frac{x}{\frac{\sin x}{x}} = \boxed{0}$

0, 1

4. $\lim\limits_{x\to 0}\dfrac{\overset{0}{\left(\dfrac{1}{x+1}-1\right)}}{\underset{0}{(x)}}\cdot\dfrac{(x+1)}{(x+1)} = \lim\limits_{x\to 0}\dfrac{1-(x+1)}{x(x+1)}$

$= \lim\limits_{x\to 0}\dfrac{-x}{x(x+1)} = \lim\limits_{x\to 0}\dfrac{-1}{x+1} = \boxed{-1}$

5. $\lim\limits_{x\to -1^+}\dfrac{\overset{5}{x^2-2x+2}}{\underset{2}{x+3}} = \boxed{\dfrac{5}{2}}$

6. $\lim\limits_{x\to -\infty} 1000+200x^2-2x^3 = \boxed{+\infty}$

leading term dominates

7. $\lim\limits_{x\to \frac{\pi}{2}^+}\dfrac{\overset{\pi/2}{x}}{\underset{0^-}{\cos x}} = \boxed{-\infty}$