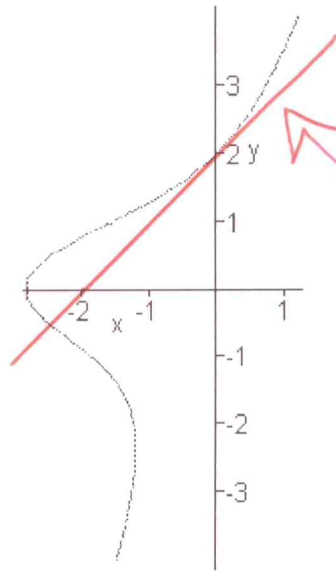
10/9/2000 | MA303 Calculus I | Name: Solution
Dr. Lunsford | Quiz 9 | (20 Points Total)

I. Find the equation of the tangent line to the graph of $x^2 - 3xy^2 + y^3 = 8$ at the point $(x, y) = (0, 2)$. A graph of the equation is given below. Graph the tangent line on the same axes as the equation. Neatly show all of your work and clearly indicate your answer. (10 points)

$$\frac{d}{dx}(x^2 - 3xy^2 + y^3) = \frac{d}{dx}8$$

$$2x - 3y^2 - 3x \cdot 2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$



At $(x,y) = (0,2)$:

$$0 - 12 - 0 + 12\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 1 \Rightarrow m = 1 \text{ at } (0,2)$$

$$y - 2 = 1(x - 0)$$

$$\boxed{y = x + 2}$$

II. Use the equation $4\sin x \cos y = 1$ to answer the following questions. (5 each, 10 total)

(a) If y is a differentiable function of x that satisfies the equation then find $\frac{dy}{dx}$.

$$\frac{d}{dx} 4\sin x \cos y = \frac{d}{dx} 1$$

$$4\cos x \sin y + 4\sin x\left(-\sin y \frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-4\cos x \sin y}{-4\sin x \sin y} = \boxed{\frac{\cos x \sin y}{\sin x \cos y}}$$

(b) Now suppose x and y are differentiable functions of t that satisfy the equation.

Find $\frac{dy}{dt}$.

$$\frac{d}{dt} 4\sin x \cos y = \frac{d}{dt} 1$$

$$4\cos x \frac{dx}{dt}\cos y + 4\sin x\left(-\sin y \frac{dy}{dt}\right) = 0$$

$$\frac{dy}{dt} = \frac{-4\cos x \cos y \frac{dx}{dt}}{-4\sin x \sin y} = \boxed{\frac{\cos x \cos y}{\sin x \sin y}\frac{dx}{dt}}$$