11/21/2011 MATH261 Calculus I Name:________________

Dr. Lunsford Test 2 (100 Points Total)

<u>Problem I.</u> Find the indicated derivatives. <u>Neatly show all of your work.</u> You DO NOT need to simplify your answers. Clearly indicate your answers: (8 points each, 40 points total)

1. $y = \ln(x)\sin^3(2x)$, $\dfrac{dy}{dx} =$

2. $w = \arctan\ u^3 + \sec\ u^2$, $\dfrac{dw}{du} =$

3. $y = \sqrt[3]{1-x^2}$, $\dfrac{dy}{dx} =$

4. $y = x^{\sin(4x)}$, $\dfrac{dy}{dx} =$

5. $f(x) = \dfrac{x^2 e^{3x}}{(x^2+1)^3}$, $f^{/}(x) = ?$

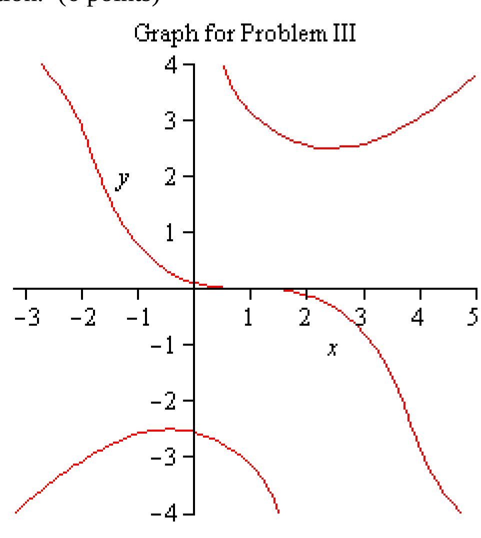
Problem II. Find the indicated limits. You must show at least one intermediate step for these limits and you must clearly show all work. If the limit does not exist as a real number but does exist in the infinite sense, please indicate the "value" of the infinite limit. (5 points each, 15 points total)

1. $\lim_{x \to 0} \frac{e^{3x} - 3x - 1}{x^2}$

2. $\lim_{t \to \infty} \left(t \ln\left(\frac{2t+1}{2t} \right) \right)$

3. $\lim_{x \to 0^+} \frac{x^3 - 1}{x^2}$

Problem III. Below you are given part of the graph of $(x-1)^3 + \pi^2 \sin(y) = (x-1)y^2$. Find and accurately draw (i.e. x and y intercepts) the tangent line to the graph at the point $(1, \pi)$. Neatly show all of your work and clearly indicate your answer. Hint: Use implicit differentiation. (6 points)



<u>Problem IV.</u> Below you are given the first and second derivate sign charts for a function. You may also assume the following about the function:

(a) The function and its derivatives are continuous on the entire real line.
(b) The function has zeros when x equals -3, 0, and 3.
(c) The first derivative of the function is zero when x equals -2, 0, and 2.
(d) The second derivative of the function is zero when x equals -1, 0, and 1.

First Derivative Sign Chart:

Interval	$-\infty, -2$	$-2, 0$	$0, 2$	$2, \infty$
Sign of $f^{/}(x)$ on the interval	Positive	Negative	Negative	Positive

Second Derivative Sign Chart:

Interval	$-\infty, -1$	$-1, 0$	$0, 1$	$1, \infty$
Sign of $f^{//}(x)$ on the interval	Negative	Positive	Negative	Positive

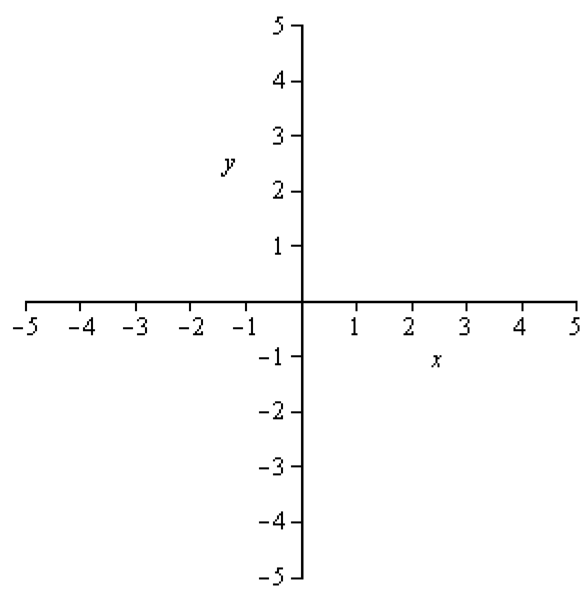
Please answer the following questions. (16 points total)

(a) <u>Use the first derivative test</u> to determine at what value(s) of x , if any, the function has relative minimum values. Explain why. (4 points)

(b) <u>Use the second derivative test</u> to determine at what value(s) of x , if any, the function has relative maximum values. Explain your reasoning. (4 points)

(c) Are there any value of x for which the second derivative test is inconclusive? Why or why not? (2 points)

(d) Graph the function showing proper concavity on the axes to your right. The function you graph should satisfy the assumptions above as well as the sign charts. Note that the the graph you draw may not be unique. (6 points)



Problem V. An 8 in^3 ice cube is pulled out of the freezer and kept at a temperature so that it melts at a constant rate of 1/2 cubic inch per minute. Assuming the ice stays in a cube while melting, Please answer the following: (12 points total)

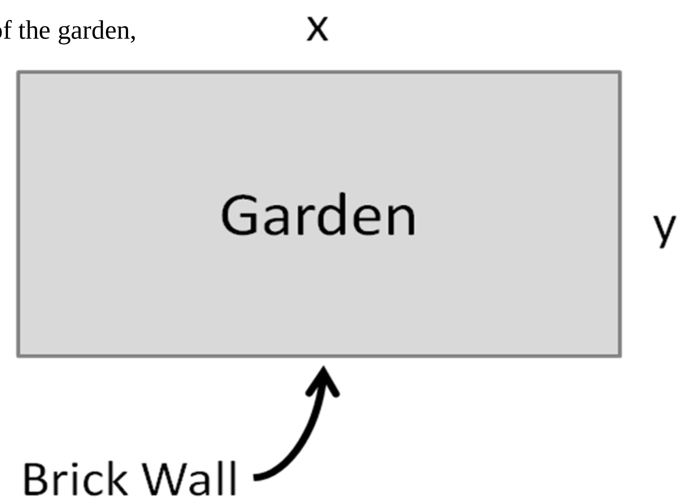
(a) Find rate of change of the side length of the cube. (4 points)

(b) How fast is the side length the cube changing when the surface area of the cube is 6 in^2? (5 points total)

(c) How long will it take the cube to melt? (3 points)

Problem VI. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall which costs $\$30/ft$ and on the other three sides by a metal fence which costs $\$10/ft$. If the area of the garden is to be $1000\,ft^2$, find the dimensions of the garden that minimizes the cost of the materials. For your viewing pleasure, a possible garden layout, including variable names, is given below. (11 points total)

(a) Find a cost function that depends on the two dimensions of the garden, $C(x, y) = ?$ (3 points)



(b) Find a relationship between x and y and use that relationship to reduce the above cost function to a function of x only, i.e., $C(x) = ?$ (3 points)

(c) Now find the value of x that minimizes the cost function you found in part (b). Be sure to clearly identify the domain over which you are finding the minimum and how you know that you have an absolute minimum on that domain. (5 points)