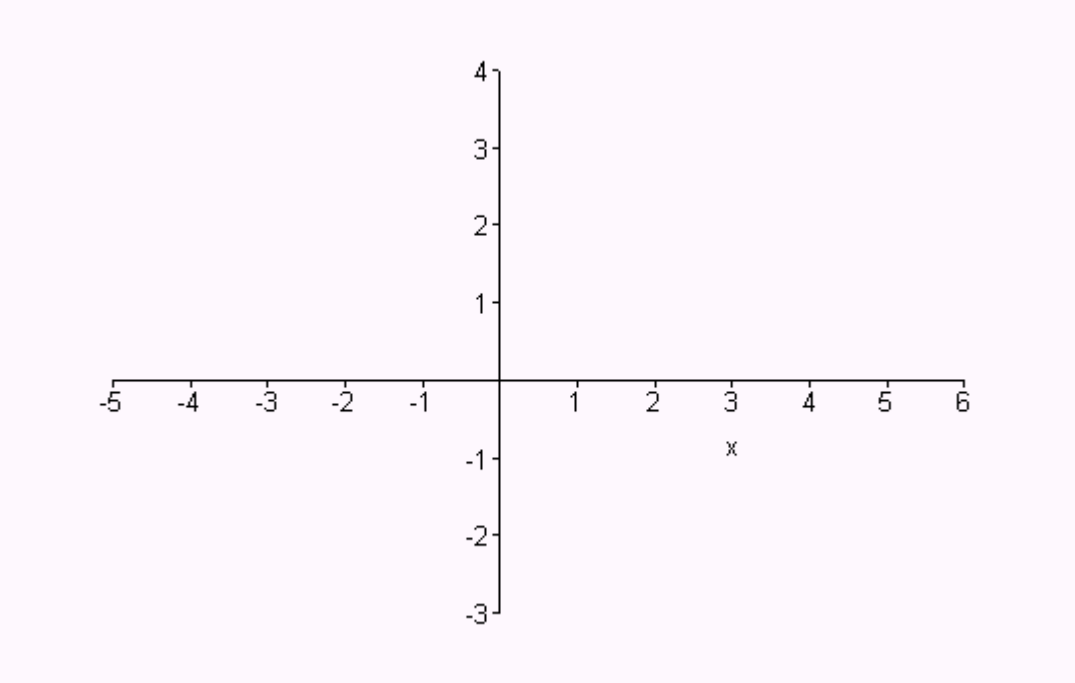
2/23/2006 MA 261 Calculus I Name:_____
Dr. Lunsford Test 1 (100 Points Total)

I. Use the graph of the function f below to answer the following questions. (2 points each – 30 total)



$f(0) =$ ___ $f(1) =$ ___ $f(-2) =$ ___

$\lim_{x \to -2} f(x) =$ ___ $\lim_{x \to 4^+} f(x) =$ ___ $\lim_{x \to 5} f(x) =$ ___

$\lim_{x \to -3^-} f(x) =$ ___ $\lim_{x \to \infty} f(x) =$ ___ $\lim_{x \to -\infty} f(x) =$ ___ $\lim_{x \to 4} f(x) =$ ___

For the remaining questions, please write "true" or "false", according to which is correct about the statement, in the space provided next to each statement.

_____ f is continuous at $x = 2$.

_____ f is continuous at $x = -2$.

_____ f is continuous on the interval $(0, 3]$

_____ f is differentiable at $x = 2$.

_____ $f'(5) > f'(1)$

II. Determine if the function given by $f(x) = \begin{cases} x^2 + 1, & x < -1 \\ 2, & x = -1 \\ -2x, & x > -1 \end{cases}$ is continuous at $x = -1$. Clearly justify your answer. (5 points)

II. For each of the following limits, determine if the limit exists as a number, exists in the infinite sense, or does not exist. If the limit exists find its value. You do not need to show any intermediate steps for these problems. Very little partial credit will be given for these problems. (3 points each, 12 total)

1. $\lim_{x\to 1^-} \frac{x}{x-1}$

2. $\lim_{x\to\infty} \frac{9-3x^2}{2x^2-2x-3}$

3. $\lim_{x\to -1} 4x^3 - 7x^{11} + 11x^5$

4. $\lim_{x\to -\infty} 4x^3 - 7x^{11} + 11x^5$

III. Consider the following table of values given below. Please answer the following questions (3 points each, 6 total)

(a) From the numerical evidence above, what is $\lim_{x\to 1} f(x)$?

x	0.9	0.99	0.995	1.005	1.01	1.1
$f(x)$	-3.013	-3.006	-3.002	-2.997	-2.991	-2.839

(b) Is it possible that $\lim_{x\to 1} f(x)$ equals some other number than your answer in part (a)? Why or why not?

IV. Let $f(x) = x^2 + 3x$. Use the limit definition of the derivative to show that $f'(x) = 2x + 3$. (10 points)

V. For each of the following limits, determine if the limit exists as a number, exists in the infinite sense, or does not exist. If the limit exists find its value. You must show at least one intermediate step to receive full credit. (5 points each – 20 points total)

1. $\lim_{u \to 1^-} \dfrac{u^2 - 6u + 5}{u^2 - 2u + 1}$

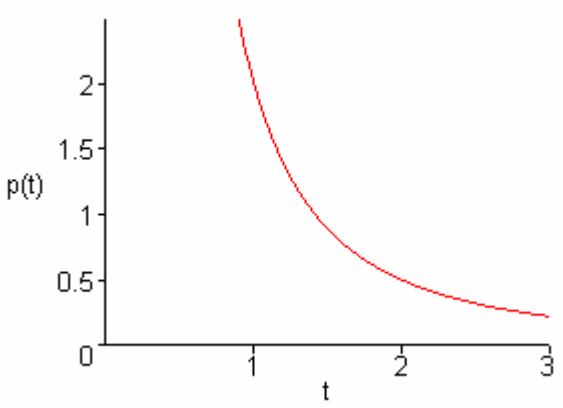
2. $\lim_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

3. $\lim_{x \to 0} x^2 \cos\left(\dfrac{1}{x} \right)$

4. $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$

VI. A particle moves along a straight path with position function given by $p(t) = \frac{2}{t^2}$ where $p(t)$ is in inches, t is in seconds and $t > 0$. A graph of this position function is given below. Please answer the following questions. (17 points total)

(a) Find the average velocity of the particle from time $t = 1$ to $t = 2$ seconds. Draw and clearly indicate the line on the graph whose slope represents this velocity. (5 points)



(b) Find the velocity of the particle at time $t = 2$ seconds. (6 points)

(c) Find the equation of the tangent line to $p(t)$ at $t = 2$. Accurately graph this line on the axes above. How is this line related to your answer for part (b)? Be sure to clearly indicate this line on the graph. (6 points)