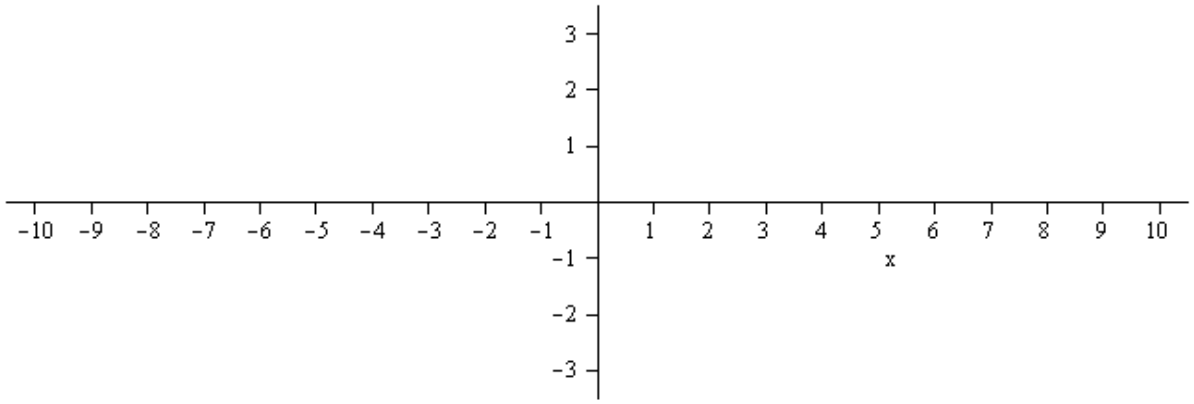
10/3/2011 MATH 261 Calculus I Name:_____
Dr. Lunsford Test 1 (100 Points Total)

For all limits on this test, determine if the limit exists as a number, exists in the infinite sense, or does not exist. If the limit exists find its value.

Problem I. Use the graph of the function f below to answer the following questions. (2 points each – 32 total)



$f(3) =$ ___ $\lim_{x\to 3} f(x) =$ ___ $\lim_{x\to -2^+} f(x) =$ ___ $\lim_{x\to -2} f(x) =$ ___

$\lim_{x\to\infty} f(x) =$ ___ $\lim_{x\to -6} f(x) =$ ___ $\lim_{x\to 1^-} \frac{x}{f(x)} =$ ___ $\lim_{x\to 0^+} f(x) =$ ___

For the remaining questions, please write "true" or "false", according to which is correct about the statement, in the space provided next to each statement.

_____ f is continuous at $x = 3$. _____ f is continuous from the left at $x = -2$.

_____ f is continuous on the interval $[-\infty, -2]$. _____ $f'(-10) < f'(-7)$.

List all x values at which f has a removable discontinuity._____

List all x values at which f is continuous but not differentiable._____

List all x values at which $f'(x) = 0$._____

Write down the equation of any vertical asymptote(s) that f may have:

Problem II. Use the function $f(x)=\begin{cases} x+1, & x\le -2 \\ x^2+1, & -2<x<2 \\ 3+x, & 2\le x \end{cases}$ to answer the following questions.

For full credit you must show at least one intermediate step for any limits you compute. (6 points total)

(a) $\lim_{x\to -1^+} f(x) =$
(2 points)

(b) Is f continuous at $x=2$? Why or why not? You must clearly justify your answer using the definition of continuity at a point. (4 points)

Problem III. Find the indicated limits – clearly indicate your answers. *You do not need to show any intermediate steps for these problems*. Very little partial credit will be given for these problems. (3 points each, 18 total)

1. $\lim_{x\to -1} 7x^{13}+9x^{14}-18x^{15} =$
2. $\lim_{x\to 1^+} \sqrt{1-x^2} =$
3. $\lim_{x\to 3^-} \dfrac{1-x}{3-x} =$
4. $\lim_{x\to -\infty} 7x^{13}+9x^{14}-18x^{15} =$
5. $\lim_{x\to \pi/4} \dfrac{\cos\ 2x}{x} =$
6. $\lim_{x\to \infty} \dfrac{7-3x^4}{6x^4+11x^2+13} =$

Problem IV. Use the limit definition of the derivative function to show that if $f(x)=\sqrt{2x+1}$ then $f'(x)=\dfrac{1}{\sqrt{2x+1}}$. (8 points)

Problem V. Find the indicated limits. You must show at least one intermediate step to receive full credit. (5 points each – 25 points total)

1. $\lim_{x\to 1^+} \dfrac{1-x^2}{x^2+7x-8}$

2. $\lim_{\theta\to 0^+} \dfrac{\sin(3\theta)}{\sqrt{\theta}}$

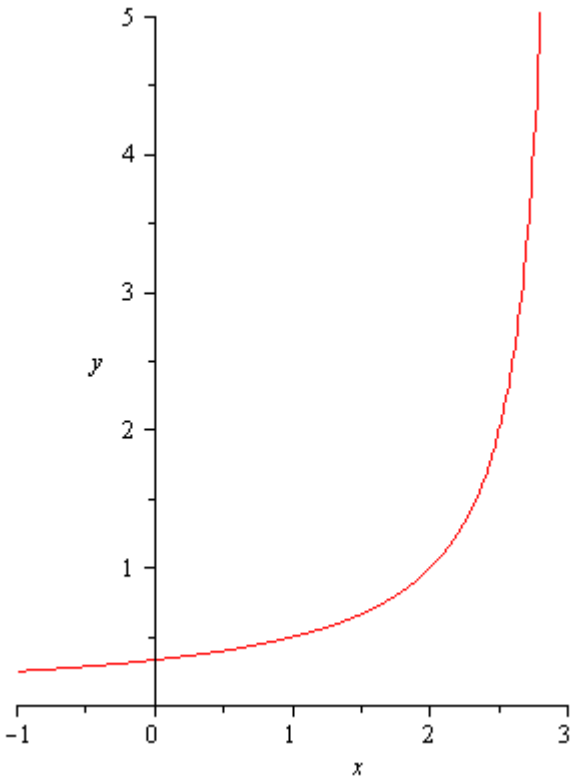
3. $\lim_{u\to 0} u^2 \sin\left(\dfrac{1}{u^2}\right)$

Hint: What is a nice theorem to use if you are between a rock and a hard place?

4. $\lim_{x\to -\infty} \dfrac{\sqrt{x^6-9}}{2x^3-6}$

5. $\lim_{x\to \frac{\pi}{2}^+} \dfrac{x+\frac{\pi}{2}}{\cos x}$

Problem VI. Below you are given part of the graph of the function $f(x)=\frac{1}{3-x}$. Please answer the following questions: (11 points total)



(a) Find the slope of the secant line through $x=0$ and $x=2$. Draw this line on the same axes as the function. (3 points)

(b) Find the slope of the tangent line to f at $x=2$. **You must use the limit definition for any credit**. (5 points)

(c) Find the equation of the tangent line to f at $x=2$ and accurately graph it on the same axes as the function. (3 points)