

Please show all work and especially show any calculator input for this test. No work or calculator input will result in no credit given (even if your answer is correct).

I. Short Answer and Multiple Choice. (45 points total)

1. Write "true" or "false" next to each statement according to which is correct. (3 points each – 9 points total)

False Keeping the sample size fixed, the margin of error for a z-confidence interval for a population mean μ will increase if the confidence level is decreased.

True At the same level of confidence, the margin of error for a z-confidence interval for a population mean μ will decrease if the sample size is increased.

False If the hypothesis test: $H_0: \mu=25$ versus $H_a: \mu \neq 25$ for a population mean μ is significant at the $\alpha = 0.05$ level, then the corresponding 95% confidence interval for μ will contain the number 25.

2. Subjects in a weight loss experiment are put on a special diet. The subjects are weighed at the beginning of the diet and weighed six weeks later at the end of the diet. Which of the following tests of significance will you use to determine if the diet was successful? (3 points)

(a) The two sample t-test.

(c) The z-test for a population proportion.

(b) The paired t-test

(d) None of the above.

3. A recent USA Today/Gallup poll reported that 61% of Americans have an unfavorable opinion of Republicans in Congress. This result was based on a sample of 1142 Americans with a margin of error of 2.3% with 95% confidence. What is the corresponding confidence interval for the true proportion of Americans that have an unfavorable opinion of Republicans in Congress? (3 points)

$$61 \pm 2.3 \Rightarrow (58.7\%, 63.3\%)$$

$$(0.587, 0.633)$$

4. Fill in the blanks: If we computed a 92% t-confidence interval for the mean age (in years) of oak trees in Farmville to be (30 years, 120 years) then the margin of error is equal to 45 years and the sample mean age is equal to 75 years. (4 points each – 8 total)

5. The P-value for a hypothesis test is $P=0.025$. At which of the following significance levels would we reject the null hypothesis? (circle one) (3 points)

(a) $\alpha = 0.10$

(b) $\alpha = 0.05$

(c) $\alpha = 0.01$

(d) all of these

(e) both (a) and (b)

Problem II. A one-sample t-test of the hypotheses $H_0: \mu = 10$ against $H_a: \mu < 10$ is performed using a sample size of 24. What is the P-value for the test if the calculated test statistic is $t = -2.13$? Draw a picture clearly illustrating both the test statistic and the P-value. (7 points)



Problem III. The SAT scores of entering freshman at University X are normally distributed with $\mu = 1200$, $\sigma = 90$. A random sample of 100 freshmen is obtained from University X, and the sample mean (i.e. average) of their 100 SAT scores is computed. What is the percent chance that this average is greater than 1217? Please be sure to show all work and all calculator input. Clearly indicate your answer. (7 points)

$$n=100 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{90}{\sqrt{100}} = 9$$

$$\text{Prob}(\bar{x} > 1217) = \text{normalcdf}(1217, 1E99, 1200, 9)$$

$$= 0.0295 \quad \boxed{2.95\% \text{ chance}}$$

Problem IV. What is the smallest sample size you would need to estimate the proportion of Americans that approve of the job Barack Obama is doing as President of the United States with a margin of error of no more than 5% with 84% confidence? Please show all work and any formulas you may use for this computation. (7 points)

$$n = \left(\frac{z^*}{m} \right)^2 p(1-p) = \left(\frac{1.405}{.05} \right)^2 (.5)(.5) = 197.4025$$

$z^*:$ $\text{invNorm}(.08) = -1.405$ 198

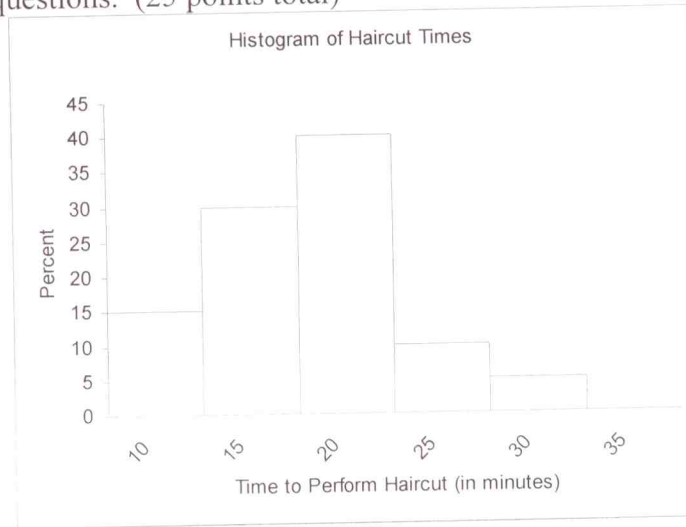
Problem VI. A real estate agent believes the mean home price of detached family homes in Prince Edward county has significantly decreased above the 2007 mean price of \$245,246. Please state the hypotheses the real estate agent should use to test her belief. Clearly indicate the population of interest, the variable measured, which hypothesis is the belief and the meaning of the population parameter used in the hypotheses in the context of this problem. (7 points)

$$H_0: \mu = \$245,246$$

$$H_a: \mu < \$245,246 \leftarrow \text{belief}$$

Pop: All detached family homes in PE county
 Var: Price of home
 μ is the mean home price for the population

Problem VI. A local barber shop wants to improve its efficiency. Let μ be the average length of time (in minutes) for a haircut at the barber shop. An efficiency expert randomly times 20 haircuts during the course of a week. Below are the data and a histogram of the data. Please answer the following questions. (25 points total)



Time to Perform Haircut (minutes)
22
19
20
20
14
17
24
21
23
18
15
26
22
19
22
14
26
12
18
34

(a) What is a point estimate for μ , the true average length of time for *all* haircuts at the barber shop? (4 points)

$$\bar{x} = 20.3$$

(b) Suppose we want to test the hypotheses

$$H_0: \mu = 24 \text{ versus } H_a: \mu < 24.$$

Which test will you use to test these hypotheses (i.e. the z-test, t-test, 1 proportion z-test, etc.) and why? (2 points)

t-test, σ unknown.

(c) What assumptions must you satisfy in order to use the test you chose in part (b). Clearly state the assumptions in the context of this problem. (6 points)

- (1) SRS from all haircuts at barbershop
- (2) Lengths of haircuts are normally distributed for all haircuts at the barbershop (or n large).

(d) Do you think the assumptions in part (c) are satisfied? You should refer to the above histogram in your answer. (3 points)

(1) Yes, given

(2) Since the histogram

of the sample is not strongly skewed and is unimodal we believe the variable for the population does not deviate too much from normality.

(e) What are the test statistic and P -value of the hypothesis test in part (b)? (6 points)

$$\text{Test Statistic} = -3.277$$

$$P\text{-value} = 0.00197$$

(f) What is your conclusion of the test in the context of the problem? (4 points)

Reject H_0 . The mean time for a haircut at this barbershop is significantly (statistically) less than 24 minutes.