10/10/2000	MA430 Theory of Prob/Stats	Name:
Dr. Lunsford	Test 1	(100 Points Total)

Neatly show all work on this test. Clearly indicate your answers. Good luck!

I. An experiment is conducted in which a fair coin is flipped three times and the outcome of each flip is recorded. Let S be the sample space, A be the event at least two of the flips are heads, and B be the event the second flip is a head. Please answer the following regarding this experiment. (20 points total)

In parts (a) - (e), determine the sets of outcomes that make up the indicated events (3 points each - 15 total):

- (a) S =
- (b) A =
- (c) B =
- (d) $A \cap B =$
- (e) $A^C \cap B^C =$

(d) Are the events A and B independent? Why or why not (i.e. justify your answer!)? (5 points)

II. Three fair dice are rolled, one red, one green, and one blue. What is the probability that the three faces are all different and that the blue die equals the sum of the red and the green dice? Note: You may assume that the sample space consists of 216 equally likely outcomes of the form (R, G, B) where R = 1, ..., 6; G = 1, ..., 6, and B = 1, ..., 6. (7 points).

III. Suppose P is a probability function on a sample space S and A and B are events in S such that

$$P(A) = 0.5, P(B) = 0.3, \text{ and } P(A \cap B) = 0.1$$

Find each of the following probabilities. (4 points each – 20 total)

- a. $P(A \cup B)$
- b. $P(B^C)$
- c. P(A | B)
- d. $P(A^C \cup B^C)$
- e. The probability that event A or event B but not both occur (i.e. the probability that at most one of the two events occurs).

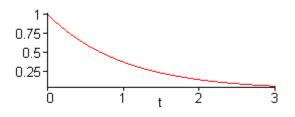
IV. An experiment is conducted in which a chip is transferred from one urn to a second urn and then a chip is drawn from the second urn. The first urn contains 3 red chips and 4 white chips and the second urn contains 3 red chips and 5 white chips.

(a) Find the probability that a red chip is ultimately drawn from the second urn. (7 points)

(b) Suppose a red chip is ultimately drawn from the second urn. What is the probability that a red chip was transferred from the first urn? (7 points)

V. The time until failure of an electrical component is given by the continuous probability function $f(t) = e^{-t}$ where *t* is in years and $0 \le t < \infty$. Find the probability that the component will last no longer than 2 years. Shade the area that represents this probability on the graph to your right. (8 points)

Graph for Problem V



VI. A fair die is repeatedly tossed until the first six appears. Please answer the following (19 points total)

- (a) What is the probability that a six appears on the first roll? (3 points)
- (b) What is the probability that the first six appears on the second roll? (3 points)
- (c) What is the probability that the first six appears on the third roll? (3 points)

(d) What is the probability that the first six appears on the n^{th} roll? (4 points)

(e) What is the probability that the first six will appear on an odd numbered roll? (6 points)

VII. A parade planner has received applications from 10 high school bands and 7 college bands. She must select 5 high school bands and 5 college bands to march in the parade.

(a) How many ways can she select the bands to march in the parade? (4 points)

(b) Once she has selected the bands, how many ways can she arrange them to march in the parade? (4 points)

(c) Suppose she wishes to alternate the high school marching bands with the college marching bands in the parade. How many ways can she now arrange the bands? Note: She can either start with a high school band or a college band in her lineup. (4 points)