Topic: Binomial Distributions and Significance Testing
Activity: Which Tire?

Example 1: Which Tire?
A legendary story on college campuses concerns two students who miss a chemistry exam because of excessive partying but blame their absence on a flat tire. The professor allowed them to take a make-up exam, and he sent them to separate rooms to take it. The first question worth five points was quite easy, and the second question worth ninety-five points asked "which tire was it?"

(a) If each student chooses a random tire, independently of each other, what is the probability that they would choose the same tire? Explain.

(b) If you were asked to identify which tire on a car had gone flat, how would you respond? Please check one response below:

_____ left front _____ right front

_____ left rear _____ right rear

(c) Pool the responses of the class; record the counts below:

left front	right front	Left rear	right rear

(d) Identify the tire that I predict will be chosen more often than "equal likeliness" would suggest.

Let the random variable X be the number of students in class who choose the tire that I predict.

(e) If I am wrong and there is nothing special about this particular tire, what type of probability distribution would X have? Explain. [Identify the parameters of the probability distribution as well as its name.]

(f) Determine the probability of obtaining at least as many "successes" as we did if there were nothing special about this particular tire. [*Hints*: Feel free to use Minitab, and remember that with a binomial distribution, $P(X \geq k) = 1 - P(X < k-1)$.]

(g) Is this probability small enough to cast doubt on the assumption that there is nothing special about this tire? Explain.

This probability is called a *p-value* of a statistical *test of significance*. If this p-value is very small, that reveals that the observed results would be very unlikely to occur by chance variation if there was nothing special about the tire in question. This in turn provides evidence against the hypothesis that the tire has probability 1/4 of being chosen in favor of the alternative that this tire is chosen more than 1/4 of the time. [*Note*: .05 is the most commonly used criterion to designate "very small," with .10 and .01 also common choices.] If this p-value is not very small, that suggests that one can not rule out chance variation as an explanation for the sample result and therefore provides little or no evidence against the hypothesis of equal likeliness. The smaller the p-value, the stronger the evidence against the "nothing special" hypothesis in favor of the alternative that people choose this tire more often than equal likeliness would suggest.

(h) How many people would have to choose this tire in order for the probability of that happening by chance, if there were nothing special about the tire, to be no greater than .05? [*Hints*: Use Minitab, first with trial-and-error until you determine the desired value. Then verify your answer with Minitab's "inverse cumulative probability option (using the value .95) under `Calc> Probability Distributions> Binomial`.]

Your calculation has identified the *rejection region* of the test, defined to be the values of the random variable that will lead you to reject the "nothing special" hypothesis.

Now suppose that the tire of interest is chosen by 30% of a sample of people.

(i) Would you find this to be convincing evidence that the long-run probability of choosing this tire is more than 1/4? What further information would you need?

(j) For the sample sizes *n* listed in the table below, find the probability of obtaining 30% or more "successes" if the long-run probability of "success" were p=1/4. [*Hint*: Use Minitab.]

sample size	20	50	100	200	500
P(30% or more)					

(k) In which of these cases would you be reasonably convinced that p>1/4? Explain.

The strength of evidence against a hypothesis depends on the *sample size* involved. Similar results provide more compelling evidence with larger sample sizes than with smaller ones.