Activity: Basic Probability Rules-Answer Key

Let A denote the subset of these 100 films that Allan has seen, so the event A = {films that Allan has seen}. Similarly define events B and F for Beth and Frank, respectively. The following 2x2 table classifies each movie according to whether it was seen by Allan and whether it was seen by Beth. The "at random" selection implies that each of the 100 films is equally likely to be chosen; i.e., each has probability 1/100. Thus, the probabilities of these various events can be calculated by counting how many of the 100 films comprise the event of interest.

	Beth yes	Beth no	Total
Allan yes	42	6	48
Allan no	17	35	52
Total	59	41	100

For example, the table reveals that 42 movies were seen by both Allan and Beth, so $P(A \cap B)=42/100=.42$ (read "the probability of A and B").

1. Translate the following events into set notation using the symbols A and B, complement, union, intersection. Also give the probability of the event as determined from the table above. Fill in the table below with these values. Lastly, draw Venn diagrams showing each of the events below.

Event in words	Event in set notation	Probability
Allan and Beth have both seen the film	$A \cap B$	$P(A \cap B)=.42$
Allan has seen the film and Beth has not	$A \cap B'$	$P(A \cap B')=.06$
Beth has seen the film and Allan has not	$A' \cap B$	$P(A' \cap B)=.17$
Neither Allan nor Beth has seen the film	$A' \cap B'$	$P(A' \cap B')=.35$

2. Fill in the marginal totals of the table (the row and column totals). From these totals determine the probability that Allan has seen a randomly selected film and also the probability that Beth has seen the film. (Remember that the film is chosen at random, so all 100 are equally likely.) Record these, along with the appropriate symbols, below.
   P(Allan has seen it) = P(A) = .48
   P(Beth has seen it) = P(B) = .59
3. Determine the probability that Allan has not seen the film. Do the same for Beth. Record these, along with the appropriate symbols, below.
   P(Allan has not seen it) = P(A') = .52
   P(Beth has not seen it) = P(B') = .41
4. If you had not been given the table, but instead had merely been told that P(A)=.48 and P(B)=.59, would you have been able to calculate P(A') and P(B')? Explain how.
   Yes we could compute the probability of the complement of an event using the probability of the event by subtracting the value of the probability of the event from one:
   P(Allan has not seen it) = P(A') = 1- P(A)=1-.48 = .52
   P(Beth has not seen it) =P(B') =1- P(B) = 1-.59= .41

One of the most basic probability rules is the *complement rule*, which asserts that the probability of the complement of an event equals one minus the probability of the event:

$$P(A') = 1-P(A)$$

5. *Add the counts in the appropriate cells of the table* to calculate the probability that at least one of Allan or Beth has seen the film (either Allan or Beth or both). Also indicate the symbols used to represent this event.
   $P(A \cup B)=.65$ (from adding 42, 6 and 17)

6. Suppose you had not been given the table but instead had merely been told $P(A)=.48$ and $P(B)=.59$ and were asked to calculate $P(A \cup B)$. You might first consider $P(A)+P(B)$. Calculate this sum and compare the result to your answer to 5). Are they the same? Is this even a legitimate answer?
   $P(A)+P(B)=1.07$ This value does not equal the value of $P(A \cup B)$ computed above. It is also not a legitimate answer since the probability function always returns a number in the interval [0,1].

7. Is $P(A)+P(B)$ larger or smaller than $P(A \cup B)$? By how much? Explain why this makes sense, and indicate how to adjust $P(A)+P(B)$ to determine $P(A \cup B)$.
   The value of $P(A)+P(B)$ is larger than the value of $P(A \cup B)$ by .42 which equals $P(A \cap B)$. This makes sense because we counted the movies in the event $A \cap B$ twice. By drawing a Venn diagram one can see that when adding the area in region A to the area in region B you count the area in region $A \cap B$ twice. Thus we need to subtract the area in region $A \cap B$. Hence we have the rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

The *addition rule* asserts that the probability of the union of two events can be calculated by adding the individual event probabilities and then subtracting the probability of their intersection: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

8. Use this addition rule as a second way to calculate the probability that Allan or Beth has seen the movie, verifying your answer to 5).
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)=.48+.59-.42=.65$

9. As a third way to calculate $P(A \cup B)$, first identify (in words and in symbols) the complement of the event {Allan or Beth has seen the movie}. Then find the probability of this complement *from the table*. Then use the complement rule to determine $P(A \cup B)$. Are your answers to 5) and 8) confirmed?
   The complement of the event {Allan or Beth has seen the movie} is the event {neither Allan nor Beth has seen the movie}. In symbols we can describe this event as $(A \cup B)'$ which equals $A' \cap B'$ via DeMorgan's Law. Thus $P(A \cup B)= 1-P(A' \cap B')=1-.35=.65$

10. What would have to be true about A and B for it to be valid to say that $P(A \cup B) = P(A) + P(B)$? *Hint*: What would the Venn Diagram look like in this case?

For $P(A \cup B) = P(A) + P(B)$ the set $A \cap B$ must be the empty set and thus $P(A \cap B)=0$. The Venn diagram would show the sets A and B with no overlap (i.e. intersection).

Two events A and B are said to be *disjoint* (or *mutually exclusive*) if their intersection is the empty set ϕ. In other words, two events are disjoint if they cannot both happen simultaneously. If $A \cap B=\phi$, then it follows that $P(A \cup B) = P(A) + P(B)$. This is known as the *addition rule for disjoint events*; it is a special case of the addition rule since if $A \cap B = \phi$, $P(A \cap B) = P(\phi) = 0$.

Three Events:

Now consider Frank, who has seen 61 of the top 100 films. The following pair of 2x2 tables (which can be considered a 2x2x2 table) reveal the counts. The 2x2 table on the left pertains to films Allan has seen, and the 2x2 table on the right pertains to films Allan has not seen:

Allan yes				Allan no			
	Frank yes	Frank no	Total		Frank yes	Frank no	Total
Beth yes	39	3	42	Beth yes	14	3	17
Beth no	3	3	6	Beth no	6	29	35
Total	42	6	48	Total	20	32	52

11. What is the probability that all three of them have seen the film? Also indicate the symbols used to denote this event.
    $P(A \cap B \cap F)=.39$

12. Express in symbols and in words the event for which the probability can be read directly from the table to be 14/100.
    The symbol for this event is: $A' \cap B \cap F$. This is the event that Allan has not seen the film and both Beth and Frank have seen the film.

We will again find three ways to calculate the probability that at least one of these three people has seen the film.

13. Express this event (that at least one of these three people has seen the film) in symbols.
    $A \cup B \cup F$

14. Determine $P(A \cup B \cup F)$ directly from the table by adding the counts of the outcomes that comprise this event.
    $P(A \cup B \cup F)=(48+14+6+3)/100=.71$

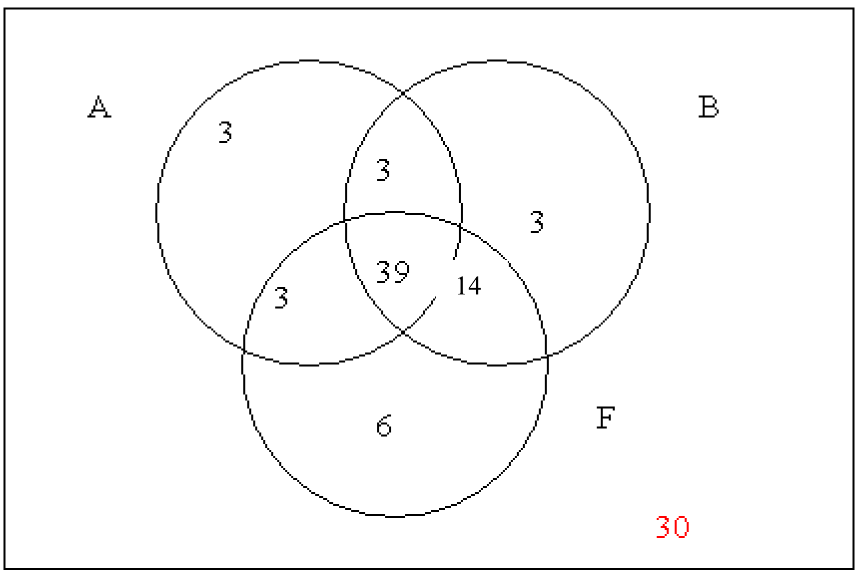
15. Express the complement of this event (that at least one of these three people has seen the film) in words and in symbols. Use the table to determine the probability of this complement. Then use the complement rule to find the probability that at least one of the three has seen the film. Does the answer agree with 14)?
    $(A \cup B \cup F)'= A' \cap B' \cap F'$ - This is the event none of these three has seen the film. From the table we have that $P(A' \cap B' \cap F')=.29$, thus $P(A \cup B \cup F)=1-P(A' \cap B' \cap F')=1-.29=.71$. This answer agrees with the answer in problem 14 above.

16. Suppose that instead of being given the tables, you had only been told that Allan has seen 48 of the films, Beth 59, and Frank 62. Would that information alone enable you to determine the number of films that at least one of these three has seen? Explain.

    This would not be enough information to determine the number of films at least one of these three has seen. This information does not tell us what films are being counted more than once (i.e. how many of Beth's 59 films coincide with Frank's 62 films).

To see how to calculate $P(A \cup B \cup F)$ from other probabilities, consider the Venn diagram below. The upper left circle represents movies that Allan has seen, the upper right circle for Beth, and the bottom circle for Frank (note that the circles and overlaps are not drawn to scale).

17. Take the eight numbers (counts) from the cells of the 2x2x2 table and insert them into the appropriate regions of the Venn diagram. For example, to get you started, the "14" representing films seen by Beth and Frank but not Allan has already been inserted.



Answers to 18 and 19 not given on this key.