Topic: Basic Probability Rules (Short)

<u>Concepts</u>: Basic Probability Rules (does not include conditional probability)

<u>Prerequisites</u>: Students should be familiar with basic set theory, sample spaces and events, and computing probabilities using the "equal likeliness" principle.

<u>Goals:</u>

- To lead students to discover some basic rules of probability: Complement rule, Addition rule for union of two events, Addition rule for disjoint events.
- To introduce students to common notation for events
- To provide experience with translating probability questions into event/set terminology and notation, and then solving the problems using the above rules.

<u>Scenario</u>: In 1998 the American Film Institute created a list of the top 100 American films ever made (www.afi.com/tv/movies.asp). Suppose that three people gather to watch a movie, and to avoid potentially endless debates about a selection, decide to choose a movie at random from the "top 100" list. You will investigate the probability that it has already been seen by at least one of the three people.

<u>Notation</u>: Let A denote the subset of these 100 films that Allan has seen, so the event A = {films that Allan has seen}. Similarly define events B and F for Beth and Frank, respectively. Recall that A' denotes the complement of A and is interpreted as "not A", that $A \cup B$ denotes the union of A and B and is interpreted as "A or B or both", and that $A \cap B$ (or simply AB) denotes the intersection of A and B and is interpreted as "A and B."

Notice that *events* are *sets*. [In particular, they are subsets of the sample space S.] Thus, it is legitimate to perform set operations such as complement, intersection, and union on them. On the other hand, *probabilities* are *numbers*. More specifically, they are numbers between 0 and 1 (including those extremes). Thus, it is legitimate to add, multiply, and divide probabilities but not to take complements, intersections, or unions of them.

The "at random" selection implies that each of the 100 films is equally likely to be chosen (i.e., each has probability 1/100). Thus, the probabilities of these various events can be calculated by counting how many of the 100 films comprise the event of interest. For example, the following 2x2 table classifies each movie according to whether it was seen by Allan and whether it was seen by Beth. It reveals that 42 movies were seen by both Allan and Beth, so $P(A \cap B) = 42/100$.

	Beth yes	Beth no	Total
Allan yes	42	6	
Allan no	17	35	
Total			100

(a) Translate the following events into set notation using the symbols A and B, complement, union, intersection. Also give the probability of the event as determined from the table:

Event in words	Event in set notation	Probability
Allan and Beth have both seen the film	$A \cap B$	$P(A \cap B)=.42$
Allan has seen the film and Beth has not		
Beth has seen the film and Allan has not		
Neither Allan no Beth has seen the film		

(b) Fill in the <u>marginal</u> totals of the table (the row and column totals). From these totals determine the probability that Allan has seen a randomly selected film and also the probability that Beth has seen the film. (Remember that the film is chosen at random, so all 100 are equally likely.) Record these, along with the appropriate symbols, below.

P(Allan has seen it) = P( ) =
P(Beth has seen it) = P( ) =

(c) Determine the probability that Allan has <u>not</u> seen the film. Do the same for Beth. Record these, along with the appropriate symbols, below.

(d) If you had not been given the table, but instead had merely been told that $P(A)=.48$ and $P(B)=.59$, would you have been able to calculate $P(A')$ and $P(B')$? Explain how.

One of the most basic probability rules is the *complement rule*, which asserts that the probability of the complement of an event equals one minus the probability of the event: $$P(A') = 1-P(A)$$

(e) Add the counts in the appropriate cells of the table to calculate the probability that either Allan or Beth (or both) have seen the movie. Also indicate the symbols used to represent this event.

(f) If you had not been given the table but instead had merely been told that $P(A) = .48$ and $P(B) = .59$, would you have been able to calculate $P(A \cup B)$? Explain.

(g) One might naively think that $P(A \cup B) = P(A) + P(B)$. Calculate this sum, and indicate whether it is larger or smaller than $P(A \cup B)$ and by how much. Explain why this makes sense, and indicate how to adjust the right side of this expression to make the equality valid.

The *addition rule* asserts that the probability of the union of two events can be calculated by adding the individual event probabilities and then subtracting the probability of their intersection: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(h) Use this addition rule as a second way to calculate the probability that Allan or Beth has seen the movie, verifying your answer to (e).

(i) As a third way to calculate this probability, first identify (in words and in symbols) the complement of the event {Allan or Beth has seen the movie}. Then find the probability of this complement from the table. Then use the complement rule to determine $P(A \cup B)$. Are your answers to (e) and (h) confirmed?

(j) What has to be true about A and B for it to be valid to say that $P(A \cup B) = P(A) + P(B)$? What would the Venn Diagram look like in this case?

Two events A and B are said to be *disjoint* (or *mutually exclusive*) if their intersection is the empty set ϕ. In other words, two events are disjoint if they cannot both happen simultaneously. If $A \cap B=\phi$, then it follows that $P(A \cup B) = P(A) + P(B)$. This is known as the *addition rule for disjoint events*; it is a special case of the addition rule since if $A \cap B = \phi$, $P(A \cap B) = P(\phi) = 0$.