4/15/2010
Dr. Lunsford

MA371 Intro. to Prob. & Stats.
Quiz 9 – The Last Quiz

Name: Solution
(40 Points Total)

Neatly show all work on this quiz. You may use your distribution sheet for this quiz.

Problem I. Suppose the random variables X_1 and X_2 are independent and have Poisson distributions with means 10 and 15, respectively. Let $Y = X_1 + X_2$. Use a moment generating function technique to determine the distribution of the Y. You should clearly indicate where in your argument you use the facts that X_1 and X_2 are independent and have Poisson distributions. (10 points)

$$m_Y(t) = E[e^{tY}] = E[e^{t(X_1+X_2)}] = E[e^{tX_1}e^{tX_2}]$$

$$= E[e^{tX_1}]E[e^{tX_2}] = m_{X_1}(t)\cdot m_{X_2}(t)$$

since X_1 and X_2 are independent

$$= e^{10(e^t-1)}e^{15(e^t-1)}$$

since X_1 is Poisson ($\lambda=10$) and X_2 is Poisson ($\lambda=15$).

$$= e^{25(e^t-1)}$$

This is the m.g.f for a Poisson r.v. w/ mean $\lambda = 25$. Thus Y is Poisson with $\lambda = 25$.

Problem II. Suppose the heights of women in a certain population can be modeled by a normal distribution with mean 63 inches and standard deviation 2 inches. Heights of men in the same population can be modeled by a normal distribution with mean 68 inches and standard deviation 3 inches. If a woman and man are randomly chosen from the population what is the percent chance that the woman will be taller than the man? Neatly show all work and clearly state the meaning of all random variables used in your computations (yes, you should use some random variables!). (10 points)

X - height of randomly chosen woman $X \sim N(\mu=63, \sigma=2)$
Y - height of randomly chosen man $Y \sim N(\mu=68, \sigma=3)$

$P(X>Y) = P(X-Y>0)$

Let $W = X - Y$. Since X and Y are independent (man, woman chosen randomly)
then $W \sim N(\mu = 63-68, \sigma^2 = 4+9=13)$ ($\sigma = \sqrt{13}$)

$\therefore P(W>0) = \text{normalcdf}(0, 1E99, -5, \sqrt{13})$

$= 0.0828$ $\boxed{8.3\% \text{ chance}}$

Problem III. Suppose that scores on the SAT Math exam can be modeled with a normal distribution with mean 515 and standard deviation 95. Ms. Epsilon teaches a high school mathematics class that has 16 students. Please answer the following neatly showing all work and clearly stating the meaning of all random variables you use: (20 points total)

(a) What is the probability that a randomly chosen student from Ms. Epsilon's class will score higher than 570 on the SAT Math exam? (7 points)

X - score of randomly chosen student, $X \sim N(\mu = 515, \sigma = 95)$

$P(X > 570) = \text{normalcdf}(570, 1E99, 515, 95)$

$\boxed{= 0.2813}$

(b) What is the probability that the average SAT Math score of the students in Ms. Epsilon's class is greater than 570? What important (and potentially questionable in this case) assumption are you making to compute this probability? (8 points)

Assume students from ms. Epsilon's class are a random sample of all students who took the SAT math exam.

We want $P(\bar{X} > 570)$, Now $\bar{X} \sim \text{normal}\left(\mu = 515, \sigma = \frac{95}{\sqrt{16}} = 23.75\right)$

So $P(\bar{X} > 570)$

$= \text{normalcdf}(570, 1E99, 515, 23.75) = \boxed{0.01}$

(c) Suppose Ms. Epsilon's class had an average score of 570 on the SAT Math test. Do you think Ms. Epsilon has bragging rights about her students' mathematical ability? Do you think Ms. Epsilon has bragging rights about her teaching ability? Why or why not? You should use the probability computed in part (b) to justify your answer. (5 points)

Assuming her students are a representative sample of all students who took the SAT math exam then ms. Epsilon has bragging rights since there is only a 1% chance of her students getting an average of 570 or higher on the exam. However, if the students are not representative, i.e. they are students from an advanced math class, then she may not have bragging rights about her teaching ability.