

Neatly show all work on this quiz.

Problem I. Suppose the random variables X and Y have joint probability mass function given by

$$f(x, y) = \frac{x+y}{21} \text{ for } x=1, 2, 3 \text{ and } y=1, 2. \text{ Please answer the following questions: (14 points total)}$$

(a) Find the marginal mass function for X (i.e. find $f_X(x)$). (5 points)

$$f_X(x) = \sum_{y=1}^2 \frac{x+y}{21} = \frac{x+1}{21} + \frac{x+2}{21} = \frac{2x+3}{21}, x=1, 2, 3$$

(b) Find the marginal mass function for Y (i.e. find $f_Y(y)$). (5 points)

$$f_Y(y) = \sum_{x=1}^3 \frac{x+y}{21} = \frac{1+y}{21} + \frac{2+y}{21} + \frac{3+y}{21} = \frac{6+3y}{21} = \frac{2+y}{7}, y=1, 2$$

(c) Are X and Y independent? Why or why not? (4 points)

No b/c $f_X(x)f_Y(y) \neq f(x, y)$ for all x and y .

OR
consider $x=1, y=1$ $f(x, y) = \frac{2}{21}$, $f_X(x)f_Y(y) = \frac{5}{21} \cdot \frac{9}{21} = \frac{45}{21^2} \neq f(x, y)$

Problem II. Suppose X_1 and X_2 are independent random variables with standard deviations of 3 and 4, respectively, and with $E[X_1] = 5$ and $E[X_2] = -7$. Let $Y = \frac{X_1 + X_2}{2}$. Find each of the following.

Please show at least one intermediate step or give a logical reason for your answer. (16 points total)

(a) $\text{cov}(X_1, X_2) = 0$ since X_1 and X_2 are independent
(3 points)

(b) $E[Y] = \frac{1}{2} [E[X_1] + E[X_2]] = \frac{1}{2} (5 + (-7)) = -1$
(4 points)

(c) The standard deviation of Y . $\text{Var}(Y) = \text{Var}\left(\frac{1}{2}(X_1 + X_2)\right)$
(4 points)

X_1 and X_2 indep. $\Rightarrow \frac{1}{4} (\text{Var}(X_1) + \text{Var}(X_2)) = \frac{1}{4} (9 + 16) = \frac{25}{4}$
(d) $E[(X_1 + X_2)^2] = E[X_1^2] + 2E[X_1]E[X_2] + E[X_2^2]$ $\therefore \sigma_Y = \frac{5}{2}$
(5 points)

$$= 34 + 2(5)(-7) + 65$$

$$= 29$$

$$E[X_1^2] = \sigma_{X_1}^2 + \mu_{X_1}^2 = 9 + 25 = 34$$

$$E[X_2^2] = \sigma_{X_2}^2 + \mu_{X_2}^2 = 16 + 49 = 65$$

Sorry about that

Problem III. Suppose the random variables X and Y have a joint probability density function given by

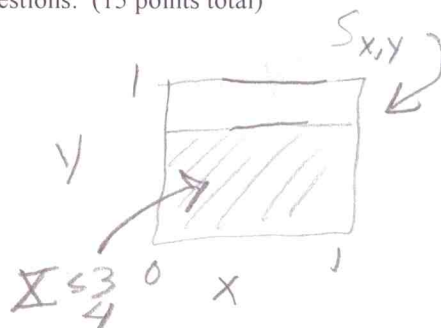
$f(x,y) = x+y$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Please answer the following questions: (15 points total)

$f(x,y)$

(a) Find $P\left(X < \frac{3}{4}\right) = \int_0^{\frac{3}{4}} \int_0^1 x+y \, dy \, dx$

(5 points)

$$= \frac{21}{32}$$



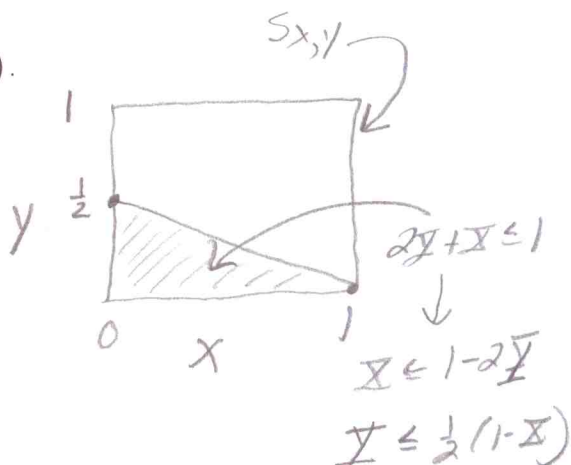
(b) Set up the integral (DO NOT INTEGRATE) to find $P(2Y + X \leq 1)$.

(5 points)

$$\int_0^{\frac{1}{2}} \int_0^{1-2y} x+y \, dx \, dy$$

OR

$$\int_0^1 \int_0^{\frac{1}{2}(1-x)} x+y \, dy \, dx$$



(c) Find $E[Y]$

(5 points)

$$\int_0^1 \int_0^1 y(x+y) \, dx \, dy = \int_0^1 \int_0^1 xy + y^2 \, dx \, dy$$

$$= \int_0^1 \left[\frac{1}{2}x^2y + xy^2 \right]_{x=0}^1 \, dy = \int_0^1 \left[\frac{1}{2} + y^2 \right] \, dy$$

$$= \left[\frac{y^2}{4} + \frac{y^3}{3} \right]_0^1 = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$