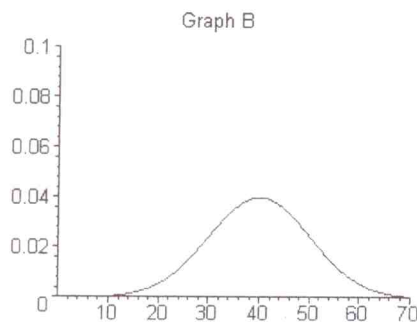
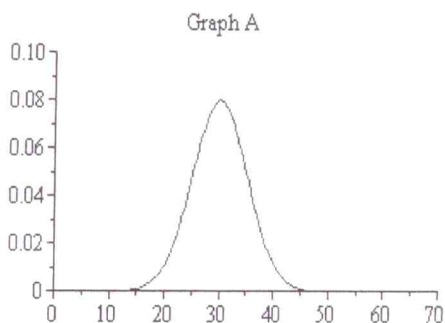


Please show all work on this quiz. IF you use your calculator to find probabilities for known distributions, then give ALL calculator input including the distribution key you used.

Problem I. Below you are given the graphs of two normal density curves. Use these density curves to answer the following questions: (5 points total)



- (a) Which curve has the larger standard deviation? B (1 point)
 (b) Which curve has the smaller mean? A (1 point)
 (c) Which distribution has a larger percent of its data between 30 and 40 units? A (1 point)
 (d) Give a rough estimate of the standard deviation of the density curve in Graph B: (2 points)

≈ 10

Problem II. A geometric random variable with probability of success p has moment generating function

$$M(t) = \frac{pe^t}{1-(1-p)e^t}. \text{ Given that } M'(t) = \frac{pe^t}{(1-(1-p)e^t)^2}, \text{ find the variance of the random variable.}$$

(10 points)

$$\sigma^2 = m''(0) - (m'(0))^2$$

$$m''(t) = \frac{pe^t (1-(1-p)e^t)^2 - pe^t (2(1-(1-p)e^t)(-(1-p)e^t))}{(1-(1-p)e^t)^2}$$

$$m''(0) = \frac{p^2 + p^2(1-p)}{p^4} = \frac{2-p}{p^2} \quad \left| \quad m'(0) = \frac{p}{p^2} = \frac{1}{p} \right.$$

$$\sigma^2 = m''(0) - (m'(0))^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}$$

Problem III. A discrete random variable, X has moment generating function

$M(t) = 0.14 + 0.23e^{-2t} + 0.41e^{2t} + 0.22e^{4t}$. Please use this information to answer the following: (5 points each, 10 total)

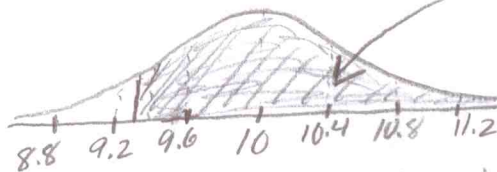
(a) Find $P(X \leq 1)$. $= P(X=0) + P(X=-2) = 0.14 + 0.23 = \boxed{0.37}$

(b) Find $E[X]$. $= m'(0) = -0.46 + 0.82 + 0.88 = \boxed{1.24}$

$$m'(t) = -0.46e^{-2t} + 0.82e^{2t} + 0.88e^{4t}$$

Problem IV. The tail length for stray cats can be modeled using a normal distribution with mean 10 inches and standard deviation 0.4 inches (Note: You might wonder how this is known!). Please use this information to answer the following questions: (25 points total)

(a) Draw a picture of the distribution of stray cat tail length showing proper concavity. Please be sure to label your horizontal axis. (6 points)



Stray Cat Tail Length (in.)

(b) Dr. L.'s stray cat, Hep Cat, has a tail that is 9.3 inches long. What is the z-score for Hep Cat's tail length? (4 points)

$$z = \frac{X - \mu_x}{\sigma_x} = \frac{9.3 - 10}{0.4} = \boxed{-1.75}$$

(c) What percent of stray cats have tails that are longer than Hep Cat's? Please represent this percent on the graph above? (5 points)

$$\text{normalcdf}(9.3, 1E99, 10, 0.4) = 0.9599$$

96%

(d) What is the range of tail lengths for cats whose tails are in the middle 50% of lengths (i.e. give the smallest and largest value for this range). (6 points)

$$\text{InvNorm}(0.25, 10, .4) = 9.73$$

$$\text{InvNorm}(0.75, 10, .4) = 10.27$$

Between 9.73 and 10.27 inches.

(e) Dr. L.'s stray cat, Kitteny Thang, has a tail length z-score of 1.35. What is Kitteny Thang's tail length? (4 points)

$$X = \mu_x + z\sigma_x = 10 + 1.35(0.4) = \boxed{10.54 \text{ inches}}$$