

I. Two fair dice are tossed, one red and one green, and the result is recorded. An example outcome is the ordered pair (2,4) which corresponds to a 2 being face up on the red die and a 4 being face up on the green die. Let  $A$  be the event that the number on the upside of the red die is a 3 and  $B$  be the event that the number on the upside of the green die is a 5. Please answer the following: (21 points total)

(a) How many outcomes are in the sample space for this random experiment (you do not need to list the outcomes)? (2 points)

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(b) Find  $P(A)$ ,  $P(A \cap B)$ , and  $P(A \cup B)$ . Clearly indicate your answers. (8 points)

$$P(A) = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\} = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = \{(3,5)\} = \frac{1}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$$

(c) Describe the event  $A' \cap B'$  using a complete English sentence. (3 points)

This is the event that the red die is not a 3 and the green die is not a 5.

(d) Let  $X$  be the random variable that assigns to each outcome of this experiment the sum of the faces of the two dice. What is  $X((2,4))$ ? (2 points)

$$2+4 = \boxed{6}$$

(e) What is  $S_X$ , the set of all possible values of  $X$  for this random experiment? (3 points)

$$S_X = \{2, 3, 4, \dots, 11, 12\}$$

(f) To what event does  $X = 4$  correspond? (3 points)

$$\{(1,3), (2,2), (3,1)\}$$

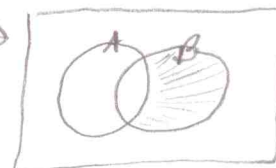
II. Suppose that  $A$  and  $B$  are events in a sample space,  $S$ , and that  $P(A) = 0.33$ ,  $P(B) = 0.54$ , and  $P(A \cup B) = 0.72$ . Find the indicated probabilities. For full credit on each problem you must show at least one intermediate step. In particular you must specifically state any set equivalences you use and/or clearly show the application of any probability laws you use. (3 points each, 12 points total)

(a)  $P(A \cap B) = P(A) + P(B) - P(A \cup B)$   
 $= .33 + .54 - .72 = \boxed{.15}$

Note:  $P(B) = P((B \cap A') \cup (B \cap A))$   
 $\Rightarrow P(B) = P(B \cap A') + P(B \cap A)$   
 $\Rightarrow P(B \cap A') = P(B) - P(B \cap A)$

(b)  $P(B') = 1 - P(B) = 1 - .54 = \boxed{.46}$

(c)  $P(A' \cap B) = P(B) - P(A \cap B)$   
 $= .54 - .15 = \boxed{.39}$



(d)  $P(A' \cap B') = P((A \cup B)')$   
 $= 1 - P(A \cup B) = 1 - .72 = \boxed{.28}$

III. When Dr. L. was in graduate school she had a cat named The Pod who had five kittens: two orange tabbies, one tortoise colored (i.e. a tortie), and two solid black. Dr. L. and Dr. P. decided they would keep two of the kittens. Please answer the following questions: (7 points total)

(a) Assuming equally likely outcomes, what is the probability that they kept an orange tabby and the tortie? To get full credit you should show the outcomes of your sample space (pictorially or list) and circle or list those outcomes that correspond to the event. Clearly indicate your answer. (5 points)

Can't Order:

	$O_1$	$O_2$	$T_1$	$B_1$	$B_2$
$O_1$	X	$O_1 O_2$	$O_1 T_1$	$O_1 B_1$	
$O_2$	$O_1 O_2$	X	$O_2 T_1$		
$T_1$	$T_1 O_2$	$T_1 O_2$	X		
$B_1$				X	
$B_2$					X

25 - 5 = 20 outcomes  
4 outcomes in event  
 $\frac{4}{20} = \frac{1}{5}$

Do not count order:

$O_1 O_2$	
$O_1 T_1$	$O_2 T_1$
$O_1 B_1$	$O_2 B_1$
$O_1 B_2$	$O_2 B_2$
$T_1 B_1$	
$T_1 B_2$	
$B_1 B_2$	

10 outcomes  
2 in event  
 $\frac{2}{10} = \frac{1}{5}$

Using counting techniques (not necessary for this quiz):  
 $\frac{\binom{2}{1} \binom{1}{1}}{\binom{5}{2}} = \frac{2}{10} = \frac{1}{5}$   
 # ways to get orange kitten  
 # ways to get tortie  
 # outcomes

(b) If Dr. L. had a preference for choosing the tortie, why would the probability you computed in part (a) not be correct? (2 points)

Because she would be more likely to choose the tortie, the outcomes of the experiment would no longer be equally likely.