

Suppose the discrete random variable  $X$  has probability mass function (p.m.f.) given by

$$f(x) = \frac{4-x}{6}, x = 1, 2, 3. \text{ Use this information to answer the following questions.}$$

(a) Draw a bar graph of the p.m.f. of  $X$ . Clearly label your axes. (3 points)



X	1	2	3
f(x)	3/6	2/6	1/6

(b) Find the probability that  $X$  is at most two. Clearly write this probability in terms of the random variable. Also show how you use the p.m.f. to compute this probability. (3 points)

$$P(X \leq 2) = P(X=1) + P(X=2) = f(1) + f(2) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

or

$$P(X \leq 2) = 1 - P(X=3) = 1 - \frac{1}{6} = \frac{5}{6}$$

(c) Find the mean of the distribution for  $X$ . Clearly show all work including what formula you are using (and how you are plugging into that formula) for this computation. (4 points)

$$\mu = \sum x f(x) = 1\left(\frac{3}{6}\right) + 2\left(\frac{2}{6}\right) + 3\left(\frac{1}{6}\right)$$
$$= \frac{10}{6} = \frac{5}{3}$$

(d) Find  $E[X^2]$ . Clearly show all of your work. (3 points)

$$E[X^2] = \sum x^2 f(x) = 1^2 \cdot \frac{3}{6} + 2^2 \cdot \frac{2}{6} + 3^2 \cdot \frac{1}{6}$$
$$= \frac{3}{6} + \frac{8}{6} + \frac{9}{6} = \frac{20}{6} = \frac{10}{3}$$

(e) Find the standard deviation of the distribution for  $X$ . Clearly show all work including what formula you are using (and how you are plugging into that formula) for this computation. (4 points)

$$\sigma^2 = E[X^2] - \mu^2 = \frac{10}{3} - \frac{25}{9} = \frac{5}{9} \Rightarrow \sigma = \frac{\sqrt{5}}{3}$$

or

$$\sigma^2 = E[(X-\mu)^2] = \left(1-\frac{5}{3}\right)^2 \cdot \frac{3}{6} + \left(2-\frac{5}{3}\right)^2 \cdot \frac{2}{6} + \left(3-\frac{5}{3}\right)^2 \cdot \frac{1}{6} = \frac{5}{9}$$

(f) Suppose we define a new random variable  $Y$  by  $Y = -2X + 7$ . Find the mean and variance of  $Y$ . Clearly indicate your answers. (3 points)

$$\mu_Y = E[Y] = E[-2X + 7] = -2E[X] + 7$$
$$= -2\mu_X + 7 = -2\left(\frac{5}{3}\right) + 7 = \frac{11}{3}$$

$$\sigma_Y^2 = \text{Var}(Y) = (-2)^2 \text{Var}(X) = 4\left(\frac{5}{9}\right) = \frac{20}{9}$$

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of text.