

Please answer the following questions, showing all intermediate steps.

I. Dr. L.'s Longwood Freshman Seminar (LSEM) class consists of 6 mathematics majors, 11 computer science majors, and 1 undecided major. She will choose a group of 4 of these students to represent the class in the upcoming LSEM Quiz Bowl. Please answer the following: (10 points total)

(a) How many possible Quiz Bowl teams can be formed? (3 points)

Choosing from 18 distinct students. Order does not matter (i.e. team of Susie, Joey, Sam, & Pam the same team regardless of order of members in list).  

$$\binom{18}{4} = 18C4 = \frac{18!}{4!14!} = 3060$$

(b) How many possible Quiz Bowl teams can be formed if one of the team members is designated to be captain of the team? (3 points)

Two ways to compute:

(a)  $\binom{18}{4} (4) = \frac{18!}{3!14!} (4)$   
 1 choose captain  
 2 choose team

(b)  $\binom{18}{1} \binom{17}{3} = \frac{18!}{3!14!}$   
 1 choose captain  
 2 choose rest of team

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Note: If compute probability w/ ordered outcomes  $\frac{\binom{18}{4} (4) (11) (3)}{(18P4)} = \dots$

(c) What is the probability that the Quiz Bowl team will consist of one mathematics major and three computer science majors? (4 points)

# ways to get math. major

$\binom{6}{1} \binom{11}{3} \binom{1}{0}$   
 # ways to get "CS" major  
 # ways to get zero "undecided"

$\binom{18}{4}$  ← # outcomes (possible teams)  
 assume equally likely

$$= \frac{6(165)}{3060} = .3235$$

II. A fair four-sided die is rolled seven times and the rolls are recorded. An example of an outcome from this random experiment is 1321342. Please answer the following questions (10 points total)

(a) How many outcomes are possible for this random experiment? (3 points)

4 possibilities for each roll, count orders:  $4^7 = 16384$

(b) How many of the outcomes of this random experiment will result in rolling a 3 exactly two times (note that the example outcome given above is one of these outcomes)? (4 points)

$\underline{3} \quad \underline{3} \quad \dots$   
 place 3's  
 $\binom{7}{2} 3^5 = 21 \cdot 3^5 = 5103$   
 4 possibilities for each of remaining slots is 3 (i.e. 1, 2, or 4)

(c) What is the probability that none of the seven rolls will be a 3? (3 points)

$\underline{3} \cdot \underline{3} \cdot \dots \cdot \underline{3}$   
 no 3 in roll (i.e. 1, 2, or 4)  
 # choices for each slot  
 counting different orders.  

$$\frac{3^7}{4^7} = \left(\frac{3}{4}\right)^7 = .1335$$
  
 equally likely outcomes