Pledge:

12/2/2004 MATH371 – Intro to Prob/Stats Name:________________
Dr. Lunsford Test 2 (100 Points Total)

Neatly show all work on this test. You may use the front flap and the appendices/tables in the back of your textbook. Clearly indicate your answers. Good luck!

I. Multiple Choice Problems. Circle the best answer for each problem. (4 points each – 16 total)

1. If the random variable X has the continuous cumulative distribution function (c.d.f.)

$$F(x) = \begin{cases} 0, & x < 0 \\ x^2/9, & 0 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

then $P(2 \le X \le 4) =$

(a) 16/9 (b) 4/3 (c) 4/9 (d) 5/9 (e) 2/3

2. Suppose Z is standard normal. Then $P(|Z| > 2.53) =$

(a) .9943 (b) .0114 (c) .0057 (d) .9886 (e) None of these

3. If Z is $N(\mu = 0, \sigma^2 = 1)$ then the value of c such that $P(|Z| < c) = .7994$ is

(a) 1.28 (b) 0.84 (c) 1.65 (d) 2.33 (e) None of these

4. If $X \sim b(n = 12, p = .45)$ then $P(X < 4) =$

(a) .3044 (b) .1345 (c) .1699 (d) .6956 (e) None of these

Problem II. The moment generating function for the Poisson distribution is given by $M(t) = e^{\lambda(e^t - 1)}$. Where λ is a constant parameter that does not depend on t. Use this moment generating function to show that the mean and the variance of the Poisson distribution both equal λ. (12 points)

Problem III. Whiskas Choice Cuts cat food comes in packages that have a label weight of 3 ounces with a standard deviation of 0.10 ounce. Let X denote the weight of a single package of Whiskas Choice Cuts cat food selected at random from a local grocery store. Assuming the weights of these packages are normally distributed, please answer the following. (20 points total)

(a) Draw a sketch of the density function of X. Your sketch should show the value of the mean and values ± 1, ± 2, and ± 3 standard deviations from the mean. (5 points)

(b) Find $P(2.845 < X < 3.054)$ and represent this probability graphically on your sketch in part (a). (6 points)

(c) Find a value of c so that $P(X > c) = 0.0885$. (4 points)

(d) Suppose 20 packages of Whiskas Choice Cuts cat food are selected independently from local grocery stores and weighed. Let Y be the number of those packages that weigh between 2.845 and 3.054 ounces. How is the random variable Y distributed? You should give the name of the distribution and the values of any relevant parameters including the possible values for Y. Hint: You should use your result from part (b). (5 points)

Problem IV. The random variables X and Y have the joint probability density function

$$f(x,y) = \begin{cases} 6x^2y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \textit{elsewhere} \end{cases}$$

Please answer the following: (14 points total)

(a) Find the marginal probability density function for X. Neatly show all of your work. (5 points)

Problem IV, continued.

(b) Given that the marginal probability density function for Y is $f_2(y) = 2y$, determine if the random variables X and Y are independent. You must justify your answer. (4 points)

(c) Set up an integral and/or a summation to find $P(Y \geq 4X^2)$. Do not compute this probability! (5 points)

Problem V. The Grinch (before he had a change of heart) decided to have some "grinchy" fun with his Christmas "gifts" one year. He carefully wrapped 20 equally sized boxes: 7 that contained ashes, 5 that contained switches, and 8 that contained bowlegged britches. He then placed the boxes in a sack and set off in the stealth of night to deliver his "gifts" to his "friends." He slinked his way down to Whoville and at the first house at which he arrived, he drew out four of the "gifts" and left them on the front doorstep. Let the random variable X denote the number of bowlegged britches among the four gifts received by this family. Please answer the following. (14 points total)

(a) How is the random variable X distributed? You should give the name of the distribution along with the values of any relevant parameters and the possible values of X. (5 points)

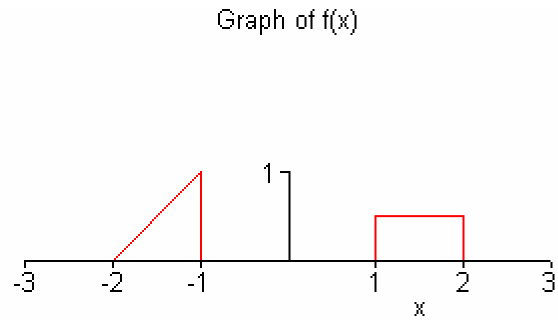
(b) What is the probability that the family will get at least one bowlegged britches? You should write this probability in terms of the random variable X as well as compute it. (5 points)

(c) On average, if this "experiment" were repeated, how many bowlegged britches would the family expect to receive from the Grinch? (4 points)

Problem VI. Let X be a random variable with probability density function

$$f(x)=\begin{cases} x+2, & -2\le x\le -1 \\ 1/2, & 1\le x\le 2 \\ 0, & elsewhere \end{cases}.$$

A graph of the p.d.f. is shown below. Please answer the following. (24 points total)



(a) Represent the probability $P\left(\frac{-4}{3}\le X\le \frac{4}{3}\right)$ on the graph of the p.d.f. (3 points)

(b) Find the cumulative distribution function (c.d.f.), $F(x)$, for X. (7 points)

(c) Find the probability in part (a) by using one of the following three methods: (i) basic geometry , (ii) using the p.d.f., (iii) using the c.d.f.. Clearly indicate which method you use and please show your work. (6 points)

(d) Find $E[X]$. (8 points)