

Neatly show all work on this quiz.

**Problem I.** Suppose the random variables  $X$  and  $Y$  have joint probability mass function given by

$f(x, y) = \frac{x+y}{21}$  for  $x = 1, 2, 3$  and  $y = 1, 2$ . Please answer the following questions: (14 points total)

(a) Find the marginal mass function for  $X$  (i.e. find  $f_X(x)$ ). (5 points)

(b) Find the marginal mass function for  $Y$  (i.e. find  $f_Y(y)$ ). (5 points)

(c) Are  $X$  and  $Y$  independent? Why or why not? (4 points)

**Problem II.** Suppose  $X_1$  and  $X_2$  are independent random variables with standard deviations of 3 and 4, respectively, and with  $E X_1 = 5$  and  $E X_2 = -7$ . Let  $Y = \frac{X_1 + X_2}{2}$ . Find each of the following. Please show at least one intermediate step or give a logical reason for your answer. (16 points total)

(a)  $\text{COV } X_1, X_2$   
(3 points)

(b)  $E Y$   
(4 points)

(c) The standard deviation of  $Y$ .  
(4 points)

(d)  $E \left[ X_1 + X_2^2 \right]$   
(5 points)

**Problem III.** Suppose the random variables  $X$  and  $Y$  have a joint probability density function given by

$f(x, y) = x + y$  for  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ . Please answer the following questions: (15 points total)

(a) Find  $P\left(X < \frac{3}{4}\right)$   
(5 points)

(b) Set up the integral (DO NOT INTEGRATE) to find  $P(2Y + X \leq 1)$ .  
(5 points)

(c) Find  $E(Y)$   
(5 points)