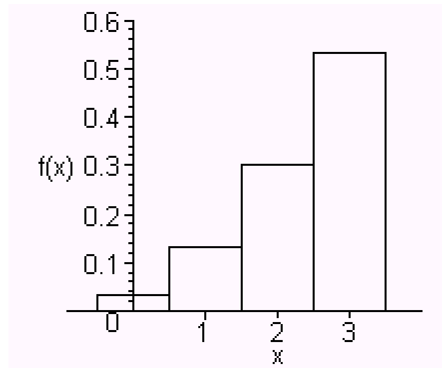
4/15/2004 MA 385 Intro. to Probability Name:________________
Dr. Lunsford Test 2 (100 Points Total)

Neatly show all work on this test. Clearly indicate your answers. You may use the front flap and tables in the appendices of your textbook. Good luck!

I. A discrete random variable X has probability mass function (p.m.f) given by $f(x) = \frac{1}{30}(x+1)^2$ for $x = 0,1,2,3$. A histogram of the p.m.f. is shown to your right. Please answer the following (8 points each, 16 points total).

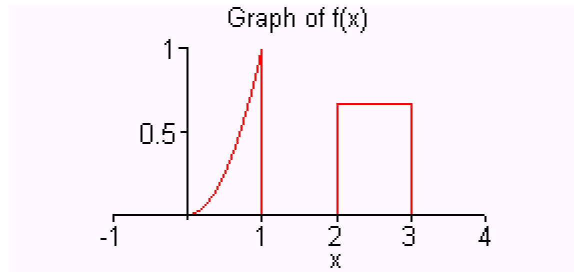


(a) Find $E[X]$ and show this value on the histogram.

(b) Find $P(X \le 1)$ and represent this probability on the histogram.

II. The Chi-Square distribution has moment generating function $M(t) = \frac{1}{(1-2t)^{r/2}}$, for $t < 1/2$ and r a constant parameter that does not depend on t. Use $M(t)$ to show that the mean of the Chi-Square distribution is r and the variance of the distribution is $2r$. (12 points)

III. A continuous random variable X has probability density function (p.d.f.) given by $f(x)=\begin{cases} x^2, & 0 \le x \le 1 \\ 2/3, & 2 \le x \le 3 \\ 0, & \textit{elsewhere} \end{cases}$. A graph of this p.d.f. is show below. Please answer the following (22 points total)



(a) Find $P\left(\frac{1}{2} \le X \le \frac{5}{2}\right)$ and show this probability on the graph of the p.d.f. (6 points)

(b) Find the cumulative distribution function (c.d.f.) for X. (8 points)

(c) Find $E[X]$ and show it on the graph of the p.d.f. (8 points)

IV. To send a "1" over a noisy transmission channel a simple repeater code is used as follows: A string of fifteen "1" bits are sent over the channel. Because of noise and other transmission errors, the probability that each bit in the string received will actually be a "1" is 0.85 and is independent of the other bits received (thus the receiver will receive a string of "1"s and "0"s of length 15). Let the random variable X be the number of "1"s in the received string. Please answer the following: (16 points total)

(a) How is the random variable X distributed? You should give the name of the distribution and the value(s) of any relevant parameter(s). (5 points)

(b) On average, how many of the bits in the received string will be "1"s? (3 points)

(c) To classify (i.e. decode) the received string as a "1", at least 8 of the bits in the string should be "1"s (otherwise the received string will be decoded as a "0"). What is the probability that a "1" sent via the repeater code will be classified as a "1"? Please be sure to write this probability in terms of the random variable X. (8 points)

V. Suppose X and Y are independent random variables with respective means μ_X and μ_Y and respective variances σ_X^2 and σ_Y^2. Let c_1 and c_2 be constants. Show that $\operatorname{var}\left(c_1 X + c_2 Y\right) = c_1^2 \sigma_X^2 + c_2^2 \sigma_Y^2$. You should clearly indicate where you use the assumption of independence in your proof. (10 points)

VI. The life of a certain type of automobile tire is normally distributed with mean 34,000 miles and standard deviation 4,000 miles. Let the random variable X be the life span in miles of a randomly chosen tire of this type. Please answer the following (24 points total)

(a) Draw a rough sketch of the distribution of X below. (4 points)

(b) What is the probability that a tire of this type will last over 40,000 miles? Please write this probability in terms of the random variable X and graph the probability on your sketch in part (a). (6 points)

(c) Find the value of m so that $P(X \geq m) = 0.10$. (6 points)

(d) Explain the meaning of the value of m found in part (c) above using a complete English sentence. (2 points)

(e) Suppose you have bought 4 randomly chosen tires of this type to put on your car. Let Y be the number of these that will last over 40,000 miles. What is the probability that all four tires will last over 40,000? Please write this probability in terms of the random variable Y and indicate any assumptions you make for this computation. Hint: You may want to take advantage of the work you have done in part (b) of this problem). (6 points)