4/15/2010
Dr. Lunsford

MA371 Intro. to Prob. & Stats.
Quiz 9 – The Last Quiz

Name: Solution
(40 Points Total)

Neatly show all work on this quiz. You may use your distribution sheet for this quiz.

Problem I. Suppose the random variables X_1 and X_2 are independent and have Poisson distributions with means 10 and 15, respectively. Let $Y = X_1 + X_2$. Use a moment generating function technique to determine the distribution of the Y. You should clearly indicate where in your argument you use the facts that X_1 and X_2 are independent and have Poisson distributions. (10 points)

$$m_Y(t) = E[e^{tY}] = E[e^{t(X_1+X_2)}] = E[e^{tX_1}e^{tX_2}]$$

$$= E[e^{tX_1}]E[e^{tX_2}] = m_{X_1}(t)\cdot m_{X_2}(t)$$

since X_1 and X_2 are independent

$$= e^{10(e^t-1)}e^{15(e^t-1)}$$

since X_1 is Poisson $(\lambda=10)$ and X_2 is Poisson $(\lambda=15)$.

$$= e^{25(e^t-1)}$$

This is the m.g.f for a Poisson r.v. w/ mean $\lambda = 25$. Thus Y is Poisson with $\lambda = 25$.

Problem II. Suppose the heights of women in a certain population can be modeled by a normal distribution with mean 63 inches and standard deviation 2 inches. Heights of men in the same population can be modeled by a normal distribution with mean 68 inches and standard deviation 3 inches. If a woman and man are randomly chosen from the population what is the percent chance that the woman will be taller than the man? Neatly show all work and clearly state the meaning of all random variables used in your computations (yes, you should use some random variables!). (10 points)

X - height of randomly chosen woman $X \sim N(\mu=63, \sigma=2)$
Y - height of randomly chosen man $Y \sim N(\mu=68, \sigma=3)$

$P(X>Y) = P(X-Y>0)$

Let $W = X - Y$. Since X and Y are independent (man & woman chosen randomly)
then $W \sim N(\mu = 63-68, \sigma^2 = 4+9 = 13)$ $(\sigma = \sqrt{13})$

$\therefore$ $P(W>0)$ = normalcdf$(0, 1E99, -5, \sqrt{13})$

$= 0.0828$ $\boxed{8.3\% \text{ chance}}$