4/8/2010 MA371 Intro. to Prob. & Stats. Name: Solution
Dr. Lunsford Quiz 8 (45 Points Total)

Neatly show all work on this quiz.

Problem I. Suppose the random variables X and Y have joint probability mass function given by

$f(x,y)=\dfrac{x+y}{21}$ for $x=1,2,3$ and $y=1,2$. Please answer the following questions: (14 points total)

(a) Find the marginal mass function for X (i.e. find $f_X(x)$). (5 points)

$$f_X(x)=\sum_{y=1}^{2}\frac{x+y}{21}=\frac{x+1}{21}+\frac{x+2}{21}=\frac{2x+3}{21},\ x=1,2,3$$

(b) Find the marginal mass function for Y (i.e. find $f_Y(y)$). (5 points)

$$f_Y(y)=\sum_{x=1}^{3}\frac{x+y}{21}=\frac{1+y}{21}+\frac{2+y}{21}+\frac{3+y}{21}=\frac{6+3y}{21}=\frac{2+y}{7},\ y=1,2$$

(c) Are X and Y independent? Why or why not? (4 points)

No b/c $f_X(x)f_Y(y)\neq f(x,y)$ for all x and y.

OR

Consider $x=1, y=1$ $f(x,y)=\frac{2}{21}$, $f_X(x)f_Y(y)=\frac{5}{21}\cdot\frac{9}{21}=\frac{45}{21^2}\neq f(x,y)$

Problem II. Suppose X_1 and X_2 are independent random variables with standard deviations of 3 and 4, respectively, and with $E[X_1]=5$ and $E[X_2]=-7$. Let $Y=\dfrac{X_1+X_2}{2}$. Find each of the following. Please show at least one intermediate step or give a logical reason for your answer. (16 points total)

(a) $\operatorname{cov}(X_1,X_2)$ $=0$ since X_1 and X_2 are independent
(3 points)

(b) $E[Y]$ $=\frac{1}{2}\left[E[X_1]+E[X_2]\right]=\frac{1}{2}(5+(-7))=-1$
(4 points)

(c) The standard deviation of Y. $Var(Y)=Var\left(\frac{1}{2}(X_1+X_2)\right)$
(4 points)

X_1 and X_2 indep. $\Rightarrow$ $=\frac{1}{4}(Var(X_1)+Var(X_2))=\frac{1}{4}(9+16)=\frac{25}{4}$ $\therefore\ \sigma_Y=\frac{5}{2}$

(d) $E\left[(X_1+X_2)^2\right]$ $=E[X_1^2]+2E[X_1]E[X_2]+E[X_2^2]$
(5 points)

$=34+2(5)(-7)+65$

$=29$

$E[X_1^2]=\sigma_{X_1}^2+\mu_{X_1}^2$
$=9+25=34$

$E[X_2^2]=\sigma_{X_2}^2+\mu_{X_2}^2$
$=16+49=65$

Sorry about that

Problem III. Suppose the random variables X and Y have a joint probability density function given by

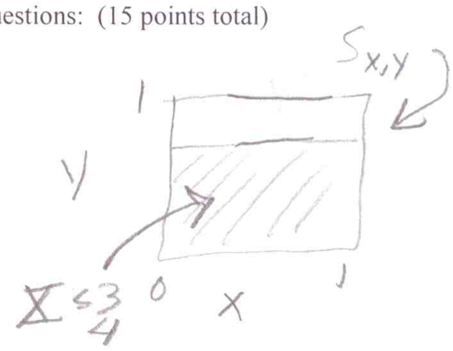
$\cancel{f(x)} = x + y$ for $0 \le x \le 1$ and $0 \le y \le 1$. Please answer the following questions: (15 points total)

$f(x,y)$

(a) Find $P\left(X < \frac{3}{4}\right) = \int_0^{3/4}\int_0^1 x+y \, dy\, dx$

(5 points)

$= \frac{21}{32}$



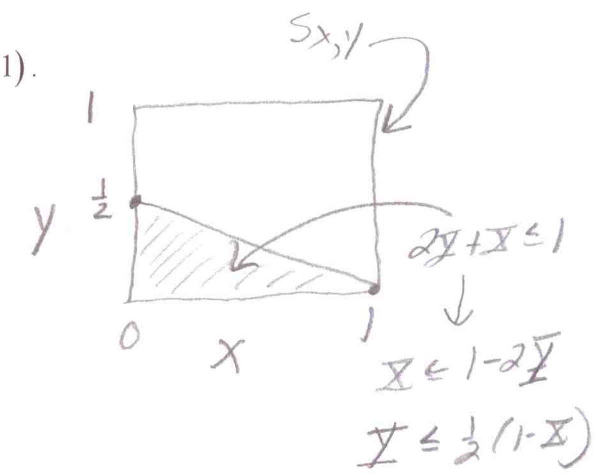
(b) Set up the integral (DO NOT INTEGRATE) to find $P(2Y + X \le 1)$.

(5 points)

$$\int_0^{1/2}\int_0^{1-2y} x+y \, dx\, dy$$

OR

$$\int_0^{1}\int_0^{\frac{1}{2}(1-x)} x+y \, dy\, dx$$



(c) Find $E[Y]$

(5 points)

$$\int_0^1\int_0^1 y(x+y)\, dx\,dy = \int_0^1\int_0^1 xy + y^2 \, dx\, dy$$

$$= \int_0^1 \frac{1}{2}x^2y + xy^2 \Big|_{x=0}^{1} dy = \int_0^1 \frac{y}{2} + y^2 \, dy$$

$$= \frac{y^2}{4} + \frac{y^3}{3}\Big|_0^1 = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$