2/22/2010 MA371 Intro. To Prob. & Stats. Name: Solution

Dr. Lunsford Quiz 5 (40 Points Total)

IF you use your calculator to find probabilities for known distributions, please give ALL calculator input including the distribution key you used.

I. The probability of a customer arrival at a fast food restaurant in any 1 second is equal to 0.08. Assume that customers arrive in a random stream and hence the arrival any one second is independent of any other. Please answer the following questions: (10 points total) X is second interval of arrival, $X = 1, 2, \ldots$

(a) Find the probability that the first arrival will occur during the third 1-second interval. (4 points) X is geometric

$$P(X=3) = (.92)(.92)(.08) = \boxed{0.0677}$$

(b) Find the probability the first arrival will not occur until at least the third 1- second interval. (4 points)

$$P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2)$$
$$= 1 - [(.08) + (.92)(.08)] = 1 - 0.1536 = \boxed{0.8464}$$

(c) On average, about how many customers would this restaurant expect to arrive in a one hour time period? (2 points)

1 hour = 3600 seconds

$$3600(.08) = \boxed{288}$$

II. Jeffery claims that thirty four percent of Americans support legalization of marijuana. You randomly select 50 Americans and ask them if they support legalization of marijuana. Assume Jeffery is correct and let the random variable X be the number in your sample who support legalization. (13 total)

(a) On average, how many people in your sample do you expect to support the legalization of marijuana if Jeffery is correct? (2 points)

$$50(.34) = \boxed{17}$$

(b) $P(X < 14)$ $= P(X \leq 13) = \text{binomcdf}(50, .34, 13)$

(4 points)

$= 0.1476$

(c) $P(X > 24)$ $= 1 - P(X \leq 24) = 1 - \text{binomcdf}(50, .34, 24)$

(4 points)

$= 0.0141$

(d) Suppose more than 24 Americans in your sample support the legalization of marijuana. Do you think Jeffery's claim is valid? Why or why not? Note: You should use the probability computed in part ~~(b)~~ (c) to answer this question.

(3 points)

If Jeffery's claim was valid there is only a 1% chance of getting more than 24 Americans in the sample. This is very unlikely hence I do not believe Jeffery's claim is valid.

III. A gambling game is played as follows: A single card is drawn from a standard deck of 52 cards. If you draw a 7 or a Jack then you win \$15.00, if you draw a 5 or a Queen then you win \$5.00, otherwise you lose \$4.00. Please answer the following: (13 points total)

(a) How much do you expect to win (or lose) per play of the game? (5 points)

$\$15.00\left(\frac{8}{52}\right) + \$5.00\left(\frac{8}{52}\right) - \$4.00\left(\frac{36}{52}\right)$

$= \$0.31$ – Win on average \$.31 per play of the game

(b) If you played the game 100 times, about how much would you expect to win (or lose)? (2 points)

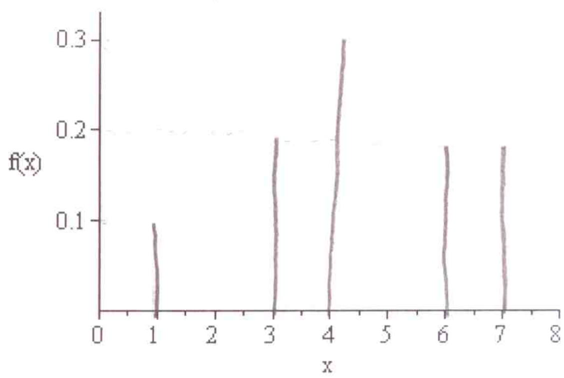
$\$0.31(100) = \$31.00$

(c) Suppose you lost y dollars instead of \$4.00 in the above scenario (i.e. when you don't draw a 5, 7, Jack, or Queen). What value of y makes this a fair game (i.e. the expected value of the game is zero)? (6 points)

$\$15.00\left(\frac{8}{52}\right) + \$5.00\left(\frac{8}{52}\right) - y\left(\frac{36}{52}\right) = 0$

$y = (15+5)\frac{8}{52} \cdot \frac{52}{36} = \$4.44$

IV. Below you are given the graph of the pmf for a discrete random variable. Find the expected value of the random variable. (4 points)



$1(.1) + 3(.2) + 4(.3)$

$+ 6(.2) + 7(.2)$

$\boxed{= 4.5}$