

I. An urn contains 4 yellow chips and 6 blue chips. A blindfolded student reaches into the urn and draws out 3 chips all at one time (i.e. without replacement). Let the random variable  $X$  be the number of yellow chips among the three chips drawn. Please answer the following questions. (10 points total)

(a) What are the number of outcomes for this random experiment? (2 points)

$$\binom{10}{3} = 10C_3 = 120$$

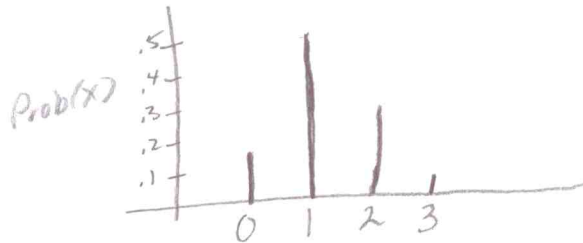
(b) Find the probability that  $X = 2$ , i.e.  $P(X = 2)$ . Show your answer in combinatorial form and compute the exact value of your answer. (4 points)

$$P(X = 2) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = \frac{3}{10} = 0.3$$

(c) Draw the distribution for the random variable  $X$ . Note that  $P(X = 0) = \frac{1}{6}$  and  $P(X = 3) = \frac{1}{30}$ .

Given this information and your answer to part (b), you should be able to easily find  $P(X = 1)$ . Please be sure to label your axes. (4 points)

$$P(X = 1) = 1 - \frac{1}{6} - \frac{1}{30} - \frac{3}{10} = \frac{1}{2}$$



II. Consider the urn in Problem I above. Now suppose that the student draws the chips one at a time and keeps track of the color of each chip drawn. Find each of the following: (4 points each, 8 total)

1. If the student draws the chips *without replacement*, then what is the probability that the first two chips drawn will be yellow and the third chip drawn will be blue?

$$\frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8} = \frac{1}{10} = 0.10$$

2. If the student draws the chips *with replacement*, then what is the probability that the first two chips drawn will be yellow and the third chip drawn will be blue?

$$\frac{4}{10} \cdot \frac{4}{10} \cdot \frac{6}{10} = \frac{96}{1000} = \frac{12}{125} = 0.096$$

III. Dr. P gave Dr. L. a hot pink ipod for her birthday a few years ago. Currently she has 200 songs stored on the ipod and likes to use the random play feature when she goes jogging with her ipod. Please answer the following. You may leave your answers in combinatorial form. (12 points total)

- (a) If the random play feature *does not allow the same song to be chosen more than once*, and 50 of the songs are jazz songs, 125 of the songs are rock and roll songs, and 25 of the songs are rap songs, then what is the probability that in the first 10 songs chosen, 3 will be jazz songs, 5 will be rock and roll songs, and 2 will be rap songs? (4 points)

$$\frac{\binom{50}{3} \binom{125}{5} \binom{25}{2}}{\binom{200}{10}}$$

- (b) If the random play feature *allows the same song to be chosen more than once* and if half of the songs are considered up-tempo songs and the other half are considered slow songs (i.e. choosing an up-tempo song is equally likely as choosing a slow song), then what is the probability that in the first 10 songs chosen, exactly 4 will be up-tempo songs? (4 points)

$$\frac{\binom{10}{4}}{2^{10}}$$

- (c) Again, if the random play feature *allows the same song to be chosen more than once*, then what is the probability that Dr. L. will have at least one song repeated in the first 10 randomly played songs? Hint: Think complement! (4 points)

$$1 - \frac{{}_{200}P_{10}}{200^{10}}$$