

I. Two fair dice are tossed, one red and one green, and the result is recorded. An example outcome is the ordered pair (2,4) which corresponds to a 2 being face up on the red die and a 4 being face up on the green die. Let A be the event that the number on the upside of the red die is a 3 and B be the event that the number on the upside of the green die is a 5. Please answer the following: (21 points total)

(a) How many outcomes are in the sample space for this random experiment (you do not need to list the outcomes)? (2 points)

36

(b) Find $P(A)$, $P(A \cap B)$, and $P(A \cup B)$. Clearly indicate your answers. (8 points)

$$P(A) = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\} = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = \{(3,5)\} = \frac{1}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$$

(c) Describe the event $A' \cap B'$ using a complete English sentence. (3 points)

This is the event that the red die is not a 3 and the green die is not a 5.

(d) Let X be the random variable that assigns to each outcome of this experiment the sum of the faces of the two dice. What is $X((2,4))$? (2 points)

$$2+4 = \boxed{6}$$

(e) What is S_X , the set of all possible values of X for this random experiment? (3 points)

$$S_X = \{2, 3, 4, \dots, 11, 12\}$$

(f) To what event does $X = 4$ correspond? (3 points)

$$\{(1,3), (2,2), (3,1)\}$$

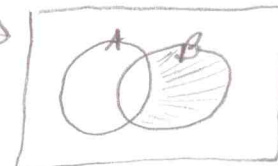
II. Suppose that A and B are events in a sample space, S , and that $P(A) = 0.33$, $P(B) = 0.54$, and $P(A \cup B) = 0.72$. Find the indicated probabilities. For full credit on each problem you must show at least one intermediate step. In particular you must specifically state any set equivalences you use and/or clearly show the application of any probability laws you use. (3 points each, 12 points total)

$$(a) P(A \cap B) = P(A) + P(B) - P(A \cup B) \\ = .33 + .54 - .72 = \boxed{.15}$$

Note: $P(B) = P((B \cap A') \cup (B \cap A))$
 $\Rightarrow P(B) = P(B \cap A') + P(B \cap A)$
 $\Rightarrow P(B \cap A') = P(B) - P(B \cap A)$

$$(b) P(B') = 1 - P(B) = 1 - .54 = \boxed{.46}$$

$$(c) P(A' \cap B) = P(B) - P(A \cap B) \\ = .54 - .15 = \boxed{.39}$$



$$(d) P(A' \cap B') = P((A \cup B)') \\ = 1 - P(A \cup B) = 1 - .72 = \boxed{.28}$$

