Pledge:

2/24/2010
Dr. Lunsford

MATH361 Calculus III
Test 1

Name: Solution
(100 Points Total)

Please show all work on this test.

Problem I. Find the angle (in degrees) between the two vectors $\mathbf{v} = \langle 2,2,1 \rangle$ and $\mathbf{u} = \langle 1,2,2 \rangle$. (5 points)

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{8}{9} \therefore \theta = 27.27^\circ$$

Problem II. Consider the points $A = (-2,1,0)$, $B = (3,3,-1)$, and $C = (0,2,0)$. Please answer the following: (14 points total)

(a) Find the equation of the plane through the three points. (10 points)

$$\vec{AB} = \langle 5,2,-1 \rangle \quad \vec{AB} \times \vec{AC} = \left| \begin{matrix} i & j & k \\ 5 & 2 & -1 \\ 2 & 1 & 0 \end{matrix} \right| = \langle 1,-2,1 \rangle$$
$$\vec{AC} = \langle 2,1,0 \rangle$$

$$1(x-0) - 2(x-2) + 1(z-0) = 0$$
$$x - 2y + z = -4$$

(b) Find the area of the triangle enclosed by the three points. (4 points)

$$\frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}|\langle 1,-2,1 \rangle| = \frac{\sqrt{6}}{2}$$

Problem III. Consider the two lines given by the following parametric equations:

Line 1: $x = 3 - t$, $y = 2 + t$, $z = 1 - t$
Line 2: $x = 3 + 2s$, $y = -2 - s$, $z = 5 + s$

Determine if the two lines are parallel, intersecting, or skew. (10 points)

slope vector line 1: $\langle -1,1,-1 \rangle$
slope vector line 2: $\langle 2,-2,1 \rangle$
} Not constant multiples of each other $\therefore$ not parallel

See if lines intersect:

$$3 - t = 3 - 2s \Rightarrow s = -\frac{t}{2}$$
$$2 - t = -2 - s \Rightarrow 2 + t = -2 + \frac{t}{2} \Rightarrow \frac{t}{2} = -4 \Rightarrow t = -8$$
$$\Rightarrow s = 4$$

check w/ z: $1-(-8) = 9$, $5+4 = 9$

point of intersection: $(11,-6,9)$

Problem IV. Let $\mathbf{u} = \langle 3, 4x+y+1, 4\rangle$ and $\mathbf{v} = \langle x+y, 2, y\rangle$. Find values of x and y such that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. Note that there is more than one possible answer here! Check to make sure the two values of x and y you find are correct. (10 points)

$\vec{u}\cdot\vec{v} = 0 \Rightarrow 3(x+y) + 2(4x+y+1) + 4y = 0$

$\Rightarrow 11x + 9y = -2$ Let $x = -1, y = 1$ then

$\vec{u} = \langle 3, -2, 4\rangle$, $\vec{v} = \langle 0, 2, 1\rangle$ and

$\vec{u}\cdot\vec{v} = 0 - 4 + 4 = 0$

Problem V. Suppose the velocity vector for an object moving in space is $\mathbf{v}(t) = \langle t^2, e^{2t}, \sin(t)\rangle$ where t is in seconds and $\mathbf{v}(t)$ is in feet per second for each component. Please answer the following: (10 points total)

(a) Find the position vector, $\mathbf{r}(t)$, for the object if $\mathbf{r}(0) = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$. (6 points)

$\vec{r}(t) = \langle \frac{1}{3}t^3 + C_1, \frac{1}{2}e^{2t} + C_2, -\cos t + C_3\rangle$

$\vec{r}(0) = \langle C_1, \frac{1}{2} + C_2, -1 + C_3\rangle = \langle 1, 4, -1\rangle$

$\Rightarrow C_1 = 1, C_2 = \frac{7}{2}, C_3 = 0$

$\therefore \vec{r}(t) = \langle \frac{1}{3}t^3 + 1, \frac{1}{2}e^{2t} + \frac{7}{2}, -\cos t\rangle$

(b) Find the acceleration vector for the object at time $t = 0$. (4 points)

$\vec{a}(t) = \vec{v}'(t) = \langle 2t, 2e^{2t}, \cos(t))$

$\vec{a}(0) = \langle 0, 2, 1\rangle$

Problem VI. Consider the surface $z = f(x,y) = x\ln(x^2 - y^3) + 3xy$. Please answer the following: (12 points total)

(a) Find $f_x(x,y)$ and $f_y(x,y)$. DO NOT simplify your answers! (8 points)

$f_x(x,y) = \ln(x^2 - y^3) + \frac{x}{x^2-y^3}\cdot 2x + 3y$

$f_y(x,y) = \frac{x}{x^2-y^3}(-3y^2) + 3x$

x fixed
y varies

(b) Find the slope of the tangent line to the curve of intersection of the surface and the plane $x = 3$, at the point $(3, 2, 36)$. (4 points)

$f_y(3,2) = \frac{-3(3)(4)}{1} + 3(3) = -27$