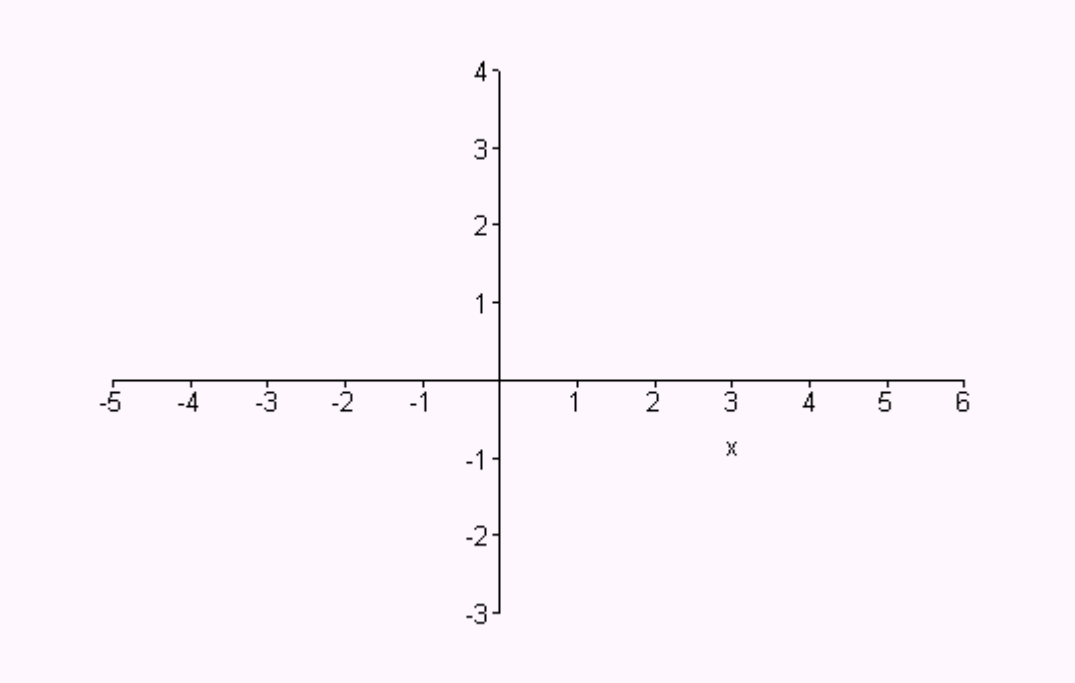
9/29/2005 MA 261 Calculus I Name:_____
Dr. Lunsford Test 1 (100 Points Total)

I. Use the graph of the function f below to answer the following questions. (2 points each – 32 total)



$f(0) =$ ___ $f(1) =$ ___ $f(3) =$ ___

$\lim_{x\to -3^-} f(x) =$ ___ $\lim_{x\to -4} f(x) =$ ___ $\lim_{x\to 3} f(x) =$ ___ $\lim_{x\to 1} f(x) =$ ___

$\lim_{x\to 3^+} f(x) =$ ___ $\lim_{x\to \infty} f(x) =$ ___ $\lim_{x\to -\infty} f(x) =$ ___ $\lim_{x\to 4} f(x) =$ ___

For the remaining questions, please write "true" or "false", according to which is correct about the statement, in the space provided next to each statement.

_____ f is continuous at $x = -2$.

_____ f is continuous at $x = 1$.

_____ f is continuous on the interval $[-3,1)$

_____ f is differentiable at $x = -2$.

_____ $f'(-4) > f'(4)$

II. Determine if the function given by $f(x) = \begin{cases} \sin x, & x < \dfrac{\pi}{4} \\ \dfrac{2\sqrt{2}}{\pi}x, & \dfrac{\pi}{4} \le x \end{cases}$ is continuous at $x = \pi/4$. Clearly justify your answer. (6 points)

III. For each of the following limits, determine if the limit exists as a number, exists in the infinite sense, or does not exist. If the limit exists find its value. You must show at least one intermediate step to receive full credit. (6 points each – 36 points total)

1. $\lim_{x\to 3}\dfrac{9-x^2}{x^2-2x-3}$

2. $\lim_{u\to 1^-}\dfrac{u^2-4u+3}{u^2-2u+1}$

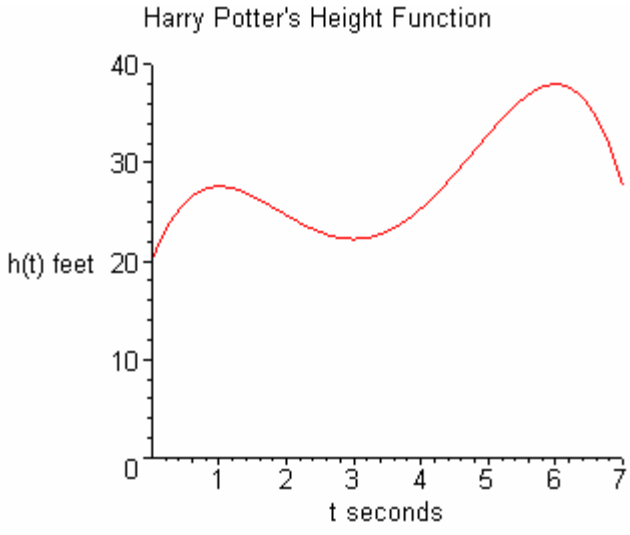
3. $\lim_{t\to -2^-}\sqrt{t^2-4}$

4. $\lim_{\theta\to -\frac{\pi}{2}^+}\dfrac{\theta+\pi}{\cos\theta}$

5. $\lim_{x\to\infty}\dfrac{3x^4-5x^2+11}{3-4x-7x^4}$

6. $\lim_{x\to 1}\dfrac{\sqrt{1+3x}-2}{x-1}$

IV. That famous fictional wizard, Harry Potter, is testing the vertical flying capability of his new Nimbus



2000 broomstick. He takes off (vertically) from the roof of Hagrid's cabin and moves only vertically (i.e. up or down). His vertical position is given by $h(t) = 20 + 18t - \frac{27}{2}t^2 + \frac{10}{3}t^3 - \frac{1}{4}t^4$ where t is measured in seconds and $h(t)$ is measured in feet. A graph of his vertical position function is given to your right. Please answer the following: (16 points total)

(a) Using the graph, briefly describe Harry's flight for the first seven seconds of the test (you should indicate the approximate time intervals when he is moving up or down and the approximate times when he changes direction). (4 points)

(b) Find Harry's average velocity from time $t = 0$ to $t = 2$ seconds. Draw and clearly indicate the line on the graph whose slope represents this velocity. (4 points)

(c) Find Harry's velocity at time $t = 2$ seconds. Draw and clearly indicate the line on the graph whose slope represents this velocity. Is Harry moving up or down at this time? Why? (4 points)

(c) Find the equation of the tangent line to $p(t)$ at $t = 2$. (4 points)

V. Let $f(x) = \frac{1}{x-2}$. Use the limit definition of the derivative to show that $f'(x) = \frac{-1}{(x-2)^2}$. (10 points)