I. Quick Integrals. Evaluate the following integrals. Very little partial credit will be given on these problems (either all, half, or nothing).   (5 points each, 25 points total)

(a) $\int \sin 3x \, dx$

(b) $\int \dfrac{1}{e^{2u}} \, du$

(c) $\int_{1}^{2} t^2 \, dt$

(d) $\int \dfrac{1}{x^2} \, dx$

(e) $\int \dfrac{1}{1-x} \, dx$

II. Below you are given the graph of a function. The area of shaded region A_1 is 3 square units and the area of shaded region A_2 is ½ square units. Also, $\int_{-2}^{4} f(x)dx = 12$. Please find the following integrals.
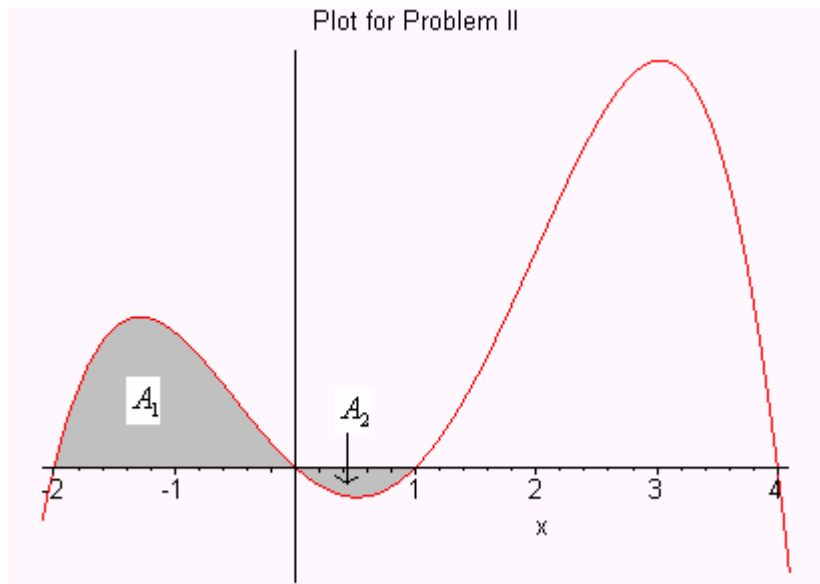
(3 points each, 12 points total)

(a) $\int_{0}^{-2} f(x) \, dx =$

(b) $\int_{-2}^{1} f(x) \, dx =$

(c) $\int_{1}^{4} f(x) \, dx =$

(d) $\int_{-2}^{1} |f(x)| \, dx =$

Plot for Problem II

III. Evaluate the following integrals. Neatly show all of your work and clearly indicate any substitutions you may make. (9 points each – 45 points total)

1. $\displaystyle\int_{0}^{1} x^2 (1+2x^3)^5 \, dx$

2. $\displaystyle\int \frac{\sqrt[3]{x} - 2x + 3}{x} \, dx$

3. $\displaystyle\int \frac{w^3}{\sqrt{1-w^2}} \, dw$

4. $\displaystyle\int (2+e^{3t})^2 \, dt$

5. $\displaystyle\int_{0}^{\pi/6} \cos^4 3x \sin 3x \, dx$

IV. Achilles and a Teenage Mutant Ninja Turtle (TMNT) are running a race. At time $t = 0$ hour they start from the same position. Achilles gets off to a great start but due to a persistent heel problem he starts to slow down. Achilles velocity function is given by $v_1(t) = \dfrac{2t}{t^2 + 2}$ in miles/hour. On the other hand, the TMNT devoured a pepperoni pizza an hour before the race and his energy levels are on the rise! The TMNT has a velocity function given by $v_2(t) = \dfrac{1}{6}t^2$ also in miles/hour. Below you are given the graphs of these two velocity functions. Please answer the following. (18 points total)

(a) What is the maximum velocity reached by Achilles during the first two hours of the race (i.e. on the time interval $[0,2]$)? At what time does he reach this maximum velocity? Clearly indicate how you are using the theory of the calculus to reach these answers. (7 points)

(b) How far ahead is Achilles at time $t = 2$ hours? Neatly show all work to justify this answer! (6 points)

(c) Represent the distance computed in part (b) on the given graph. (2 points)

(d) Make an educated guess for the time when the TMNT will catch up with Achilles. Explain your answer geometrically. (3 points)

Achilles vs. the TMNT

t TMNT
Achilles

(e) BONUS: Set up an integral equation (with unknown x) whose solution will give the exact time that the TMNT catches up with Achilles. DO NOT attempt to solve this equation!