

Please show all work and especially show any calculator input for this test. No work or calculator input will result in no credit given (even if your answer is correct).

**I. Short Answer and Multiple Choice.** (45 points total)

1. Write “true” or “false” next to each statement according to which is correct. (3 points each – 9 points total)

\_\_\_\_\_ Keeping the sample size fixed, the margin of error for a z-confidence interval for a population mean  $\mu$  will increase if the confidence level is decreased.

\_\_\_\_\_ At the same level of confidence, the margin of error for a z-confidence interval for a population mean  $\mu$  will decrease if the sample size is increased.

\_\_\_\_\_ If the hypothesis test:  $H_0: \mu = 25$  versus  $H_a: \mu \neq 25$  for a population mean  $\mu$  is significant at the  $\alpha = 0.05$  level, then the corresponding 95% confidence interval for  $\mu$  will contain the number 25.

2. Subjects in a weight loss experiment are put on a special diet. The subjects are weighed at the beginning of the diet and weighed six weeks later at the end of the diet. Which of the following tests of significance will you use to determine if the diet was successful? (3 points)

(a) The two sample t-test.

(b) The paired t-test.

(c) The z-test for a population proportion.

(d) None of the above.

3. A recent USA Today/Gallup poll reported that 61% of Americans have an unfavorable opinion of Republicans in Congress. This result was based on a sample of 1142 Americans with a margin of error of 2.3% with 95% confidence. What is the corresponding confidence interval for the true proportion of Americans that have an unfavorable opinion of Republicans in Congress? (3 points)

4. Fill in the blanks: If we computed a 92% t-confidence interval for the mean age (in years) of oak trees in Farmville to be (30 years, 120 years) then the margin of error is equal to \_\_\_\_\_ years and the sample mean age is equal to \_\_\_\_\_ years. (4 points each – 8 total)

5. The P-value for a hypothesis test is  $P=0.025$ . At which of the following significance levels would we reject the null hypothesis? (circle one) (3 points)

(a)  $\alpha = 0.10$

(b)  $\alpha = 0.05$

(c)  $\alpha = 0.01$

(d) all of these

(e) both (a) and (b)

**I. Short Answer and Multiple Choice, continued.**

6. Let  $\mu_1$  be the average yearly income of students who major in political science five years after graduating from college. A test of the hypotheses  $H_0 : \mu_1 = \$45,000$  versus  $H_a : \mu_1 > \$45,000$  resulted in a  $P$ -value of 0.23. At the  $\alpha = 0.01$  level, what can we conclude? (circle one) (3 points)

- (a) Reject  $H_0$  – five years after graduating from college political science majors earn more on average than \$45,000.
- (b) Reject  $H_0$  – five years after graduating from college political science majors do not earn more on average than \$45,000.
- (c) Fail to reject  $H_0$  – five years after graduating from college political science majors earn more on average than \$45,000.
- (d) Fail to reject  $H_0$  – five years after graduating from college political science majors do not earn more on average than \$45,000.

7. Based on sample data a researcher has calculated a 95% confidence interval for the mean lifetime,  $\mu$ , of a certain brand of light bulbs to be  $2314 \pm 18$  hours. Note this implies the confidence interval is  $2296 < \mu < 2332$ . Please write “true” or “false” next to each statement below according to which is correct: (2 points each, 6 points total)

- \_\_\_\_\_ 95% of the light bulbs lasted between 2296 and 2332 hours. (2 points)
- \_\_\_\_\_ There is a 95% chance that  $\mu$  is between 2296 and 2332 hours.
- \_\_\_\_\_ If we took many additional random samples of the same size and from each computed a 95% confidence interval for  $\mu$ , approximately 95% of these intervals would contain  $\mu$ .

8. For small samples, t intervals are (circle one)

narrower                      the same as                      wider

than z intervals based on the same data set. (3 points)

9. If I take a sample of the GPAs for 25 students in my classes and the sample standard deviation of the GPAs was 0.14 and the sample mean was 2.83, what is the standard error of the sample mean? Please show all work and any formulas you may use to get your answer. (4 points)

10. Suppose we are testing the null hypothesis  $H_0 : \mu = 50$  with the alternative  $H_a : \mu \neq 50$  for a normal population with  $\sigma = 6$ . The 95% confidence interval for the mean is (51.3, 54.7). Therefore (please circle one): (3 points)

- (a) the  $P$ -value for the test is greater than 0.05.
- (b) the  $P$ -value for the test is less than 0.05.
- (c) the  $P$ -value for the test could be greater or less than 0.05. It cannot be determined without knowing the sample size.

**Problem II.** A one-sample t-test of the hypotheses  $H_0 : \mu = 10$  against  $H_a : \mu < 10$  is performed using a sample size of 24. What is the P-value for the test if the calculated test statistic is  $t = -2.13$ ? Draw a picture clearly illustrating both the test statistic and the P-value. (7 points)

**Problem III.** The SAT scores of entering freshman at University X are normally distributed with  $\mu = 1200$ ,  $\sigma = 90$ . A random sample of 100 freshmen is obtained from University X, and the sample mean (i.e. average) of their 100 SAT scores is computed. What is the percent chance that this average is greater than 1217? Please be sure to show all work and all calculator input. Clearly indicate your answer. (7 points)

**Problem IV.** What is the smallest sample size you would need to estimate the proportion of Americans that approve of the job Barack Obama is doing as President of the United States with a margin of error of no more than 5% with 84% confidence? Please show all work and any formulas you may use for this computation. (7 points)

**Problem VI.** A real estate agent believes the mean home price of detached family homes in Prince Edward county has significantly decreased above the 2007 mean price of \$245,246. Please state the hypotheses the real estate agent should use to test her belief. Clearly indicate the population of interest, the variable measured, which hypothesis is the belief and the meaning of the population parameter used in the hypotheses in the context of this problem. (7 points)

**Problem VI.** A local barber shop wants to improve its efficiency. Let  $\mu$  be the average length of time (in minutes) for a haircut at the barber shop. An efficiency expert randomly times 20 haircuts during the course of a week. Below are the data and a histogram of the data. Please answer the following questions. (25 points total)

(a) What is a point estimate for  $\mu$ , the true average length of time for *all* haircuts at the barber shop? (4 points)

(b) Suppose we want to test the hypotheses

$$H_0: \mu = 24 \text{ versus } H_a: \mu < 24.$$

Which test will you use to test these hypotheses (i.e. the z-test, t-test, 1 proportion z-test, etc.) and why? (2 points)

(c) What assumptions must you satisfy in order to use the test you chose in part (b). Clearly state the assumptions in the context of this problem. (6 points)

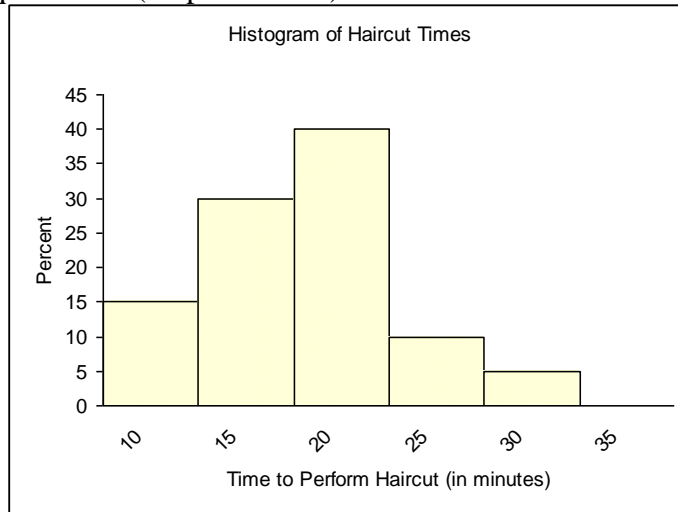
(d) Do you think the assumptions in part (c) are satisfied? You should refer to the above histogram in your answer. (3 points)

(e) What are the test statistic and  $P$ -value of the hypothesis test in part (b)? (6 points)

Test Statistic =

$P$ -value =

(f) What is your conclusion of the test in the context of the problem? (4 points)



Time to Perform Haircut (minutes)
22
19
20
20
14
17
24
21
23
18
15
26
22
19
22
14
26
12
18
34

**Problem VII.** Please circle the appropriate answer: (2 points)

- (a) I purchased a paperback textbook for this course – at the Longwood bookstore.
- (b) I purchased a paperback textbook for this course – NOT at the Longwood bookstore.
- (c) I purchased a hardback textbook for this course – at the Longwood bookstore.
- (d) I purchased a hardback textbook for this course – NOT at the Longwood bookstore.
- (e) I did not purchase a textbook because I use the online version of the textbook.
- (f) Other: Please explain