

I. An “ace-six flats” die is rolled. Let $P(i)$ be the probability that the i^{th} face appears and suppose that $P(1) = P(6) = \frac{1}{4}$ and $P(2) = P(3) = P(4) = P(5) = \frac{1}{8}$.

(a) What is the probability that the roll (i.e. the face that appears) is less than five? (4 points)

Let A be the event that the roll is less than five. Then $A = \{1,2,3,4\}$ and thus

$$P(A) = P(1) + P(2) + P(3) + P(4) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$$

(b) What is the probability that the roll is a one given that the roll is less than five? (4 points)

Let B be the event the roll is a one. Then $P(B) = \frac{1}{4}$ and the probability that the roll is a one given the

roll is less than five is $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)} = \frac{1/4}{5/8} = \frac{2}{5}$. Note that since $B \cap A = A$ we

have $P(B \cap A) = P(A)$.

II. The time it takes for a student to complete their degree at ASU has the continuous probability (density)

function $f(x) = \frac{3}{80}(6x - x^2), 0 < x < 4$ where x is in years. What is the probability that a student

will complete their degree within three years? Shade the area that represents this probability on the graph below. (NOTE: Should be 4 points, not 6 as on your original quiz)

Let A be the event a student completes their degree within three years. Then

$P(A) = \int_0^3 \frac{3}{80}(6x - x^2) = \frac{54}{80} = .675$. The area shaded should be the area bounded by the graph of the

probability function, the x-axis and the lines $x=0$ and $x=3$, i.e. the area under the curve from 0 to 3. Note: Graph not in this html document.

III. A track coach has 10 sprinters and 6 long distance runners from which to form a team of three sprinters and two long distance runners.

(a) How many teams can she form? (4 points) Note: We do not want to count a different permutation of players as a different team, thus order does not matter in our selection.

$$\binom{10}{3} \binom{6}{2} = 1800$$

(b) Suppose she wants to designate one of the sprinters as the team captain. How many teams can she now form? (4 point)

$$\binom{10}{1} \binom{9}{2} \binom{6}{2} = 10 \binom{9}{2} \binom{6}{2} = 5400$$